



A study on the **Dynamics**  
of **Interplanetary gas** and **Magnetic field**

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**Sudipta**

# Introduction

# Introduction

- Ionized comet tails always point away from the Sun
- Independent of the comet's direction of motion
- Implies a continuous outward flow from the Sun

## Key Inference:

Radial plasma outflow from the Sun

# Static Equilibrium

# Spherical Symmetry

- Flow assumed symmetric in all directions
- Reduces 3D problem to 1D radial flow

$$v = v(r), \quad N = N(r), \quad T = T(r)$$

- All physical quantities depend only on radial distance

# Observed Physical Scales

- Velocity:

$$v \sim 500 - 1500 \text{ km/s}$$

- Density near Earth:

$$n \sim 500 \text{ cm}^{-3}$$

- Mass loss rate:

$$\dot{M} \sim 10^{14} - 10^{15} \text{ g/s}$$

# Central Physical Problem

- What mechanism accelerates plasma to  $\sim 1000$  km/s?
- Known mechanisms fail to explain observed speeds

**Thermal velocity estimate:**

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v_{\text{thermal}} = \sqrt{\frac{3kT}{m}}$$

# Thermal vs Escape Velocity

- For  $T \sim 3 \times 10^6$  K:

$$v_{\text{thermal}} \approx 260 \text{ km/s}$$

$$v_{\text{escape}} \approx 500 \text{ km/s}$$

## Conclusion:

- Individual particles cannot escape
- Collective flow must be considered

# Parker's Hypothesis

- High coronal temperature drives expansion
- Pressure gradients act globally

$$\nabla P \neq 0 \Rightarrow \text{outward acceleration}$$

## Key Idea

Solar wind is a consequence of fluid dynamics, not an assumption.

# Equation of State

Fully ionized hydrogen plasma:

- One proton + one electron per atom

$$n_{\text{total}} = 2N$$

$$p = 2NkT$$

# Hydrostatic Equilibrium

Force balance:

$$-\nabla P - \rho \nabla \Phi = 0$$

In radial form:

$$\frac{dp}{dr} = -\rho \frac{GM}{r^2}$$

Using:

$$\rho = Nm$$

# Governing Equation

Using  $p = 2NkT$ :

$$\frac{d}{dr}(2NkT) = -Nm\frac{GM}{r^2}$$

Expanding:

$$2k \left( T \frac{dN}{dr} + N \frac{dT}{dr} \right) = -Nm \frac{GM}{r^2}$$

# Heat Transport

Thermal conduction:

$$\nabla \cdot (\kappa(T) \nabla T) = 0$$

For plasma:

$$\kappa(T) \propto T^{5/2}$$

- Steady-state heat flow
- No sources beyond  $r = a$

# Temperature Profile

Solution:

$$T(r) = T_0 \left( \frac{a}{r} \right)^{1/(n+1)}$$

- Power-law decay
- Controlled by heat conduction

# Density Equation

From hydrostatic equation:

$$\frac{dN}{N} = \left[ \frac{1}{(n+1)r} - \frac{GMm}{2kTr^2} \right] dr$$

# Density Profile

Solution:

$$N(r) = N_0 \left(\frac{a}{r}\right)^{1/(n+1)} \exp \left[ \frac{\lambda(n+1)}{n} \left( \left(\frac{a}{r}\right)^{n/(n+1)} - 1 \right) \right]$$

# Pressure Profile

$$p(r) = 2N(r)kT(r)$$

$$p(\infty) = p_0 \exp \left[ -\frac{\lambda(n+1)}{n} \right]$$

# Asymptotic Behaviour

- Pressure at infinity is finite:

$$p(\infty) < \infty$$

## Key Result

Hydrostatic corona requires finite pressure at infinity.

# Physical Contradiction

- Interstellar medium pressure:

$$p_{\text{ISM}} \sim 10^{-13} \text{ dyn/cm}^2$$

$$p(\infty) \gg p_{\text{ISM}}$$

## Conclusion

No static equilibrium possible.

# Final Conclusion

- Hydrostatic equilibrium leads to contradiction

The solar corona must expand outward

- Leads naturally to solar wind

# Numerical Setup-1

- Governing equation:

$$\frac{d \ln N}{dr} = \frac{1}{(n+1)r} - \frac{GMm}{2kTr^2}$$

- Temperature profile:

$$T(r) = T_0 \left( \frac{a}{r} \right)^{1/(n+1)}$$

- Numerical integration performed in logarithmic form
- Radial domain:

$$r \in [a, 10^3 a]$$

## Implementation:

[github.com] (<https://github.com/SD2189/Fluid-and-MHD-Project/blob/main/Numerical%20assignment1.ipynb>)

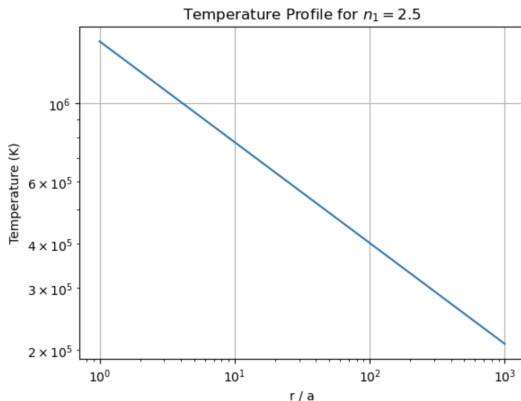
# Physical Parameters

| Parameter | Value                              |
|-----------|------------------------------------|
| $G$       | $6.67 \times 10^{-11}$             |
| $M$       | $2 \times 10^{30} \text{ kg}$      |
| $m$       | $1.67 \times 10^{-27} \text{ kg}$  |
| $k$       | $1.38 \times 10^{-23} \text{ J/K}$ |
| $a$       | $10^9 \text{ m}$                   |
| $T_0$     | $1.5 \times 10^6 \text{ K}$        |
| $N_0$     | $3 \times 10^{13} \text{ m}^{-3}$  |

$$\lambda = \frac{GMm}{2kT_0a} \approx 5.38$$

**Agreement with Parker:**  $\lambda \approx 5.35$

# Temperature Profile ( $n = 2.5$ ) | Ionized Hydrogen

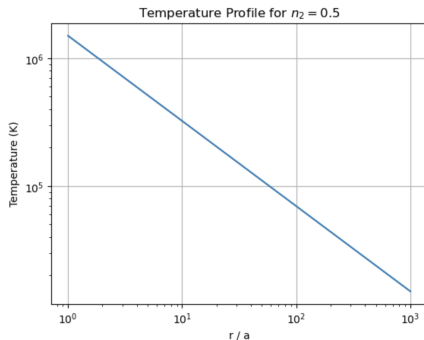


$$T(r) \propto r^{-1/(n+1)} = r^{-1/3.5}$$

## Interpretation:

- Slow decay of temperature
- Corona remains hot at large distances

# Temperature Profile ( $n = 0.5$ ) | Neutral Hydrogen

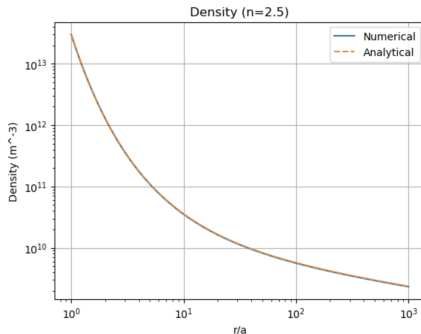


$$T(r) \propto r^{-1/1.5}$$

## Interpretation:

- Faster temperature decay
- Strong cooling of corona

# Density Profile ( $n = 2.5$ )



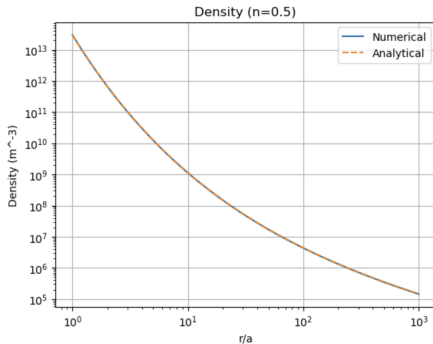
## Result:

- Numerical and analytical solutions overlap

## Conclusion:

Numerical solution validates analytical derivation

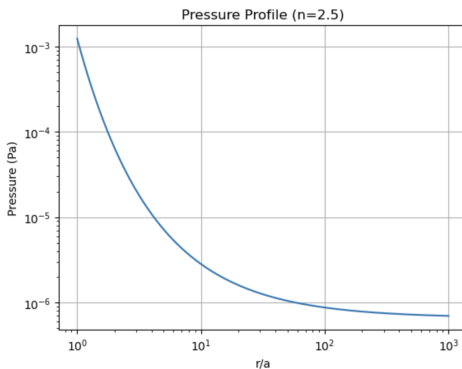
# Density Profile ( $n = 0.5$ )



## Observation:

- Excellent agreement with theory

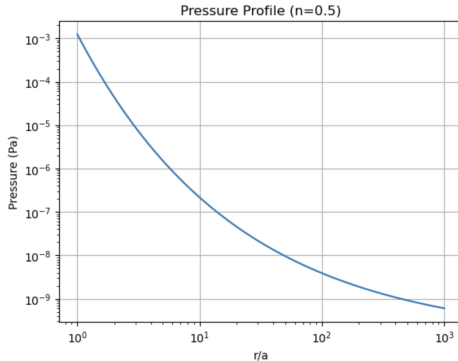
# Pressure Profile ( $n = 2.5$ )



## Key Feature:

- Pressure flattens at large  $r$   $p(\infty) \neq 0$

# Pressure Profile ( $n = 0.5$ )



## Observation:

- Faster decay but still non-zero asymptote

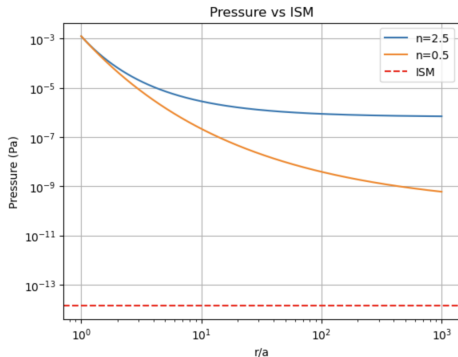
# Comparison with Theory

| Case      | Analytical $p(\infty)$    | Numerical $p(\infty)$     |
|-----------|---------------------------|---------------------------|
| $n = 2.5$ | $6.64 \times 10^{-7}$ Pa  | $6.95 \times 10^{-7}$ Pa  |
| $n = 0.5$ | $1.21 \times 10^{-10}$ Pa | $6.01 \times 10^{-10}$ Pa |

## Conclusion:

- Excellent agreement
- Minor deviation due to finite domain ( $r \neq \infty$ )

# Comparison with Interstellar Medium



$$p_{\text{ISM}} \sim 1.4 \times 10^{-14} \text{ Pa}$$

**Result:**

$$p(\infty) \gg p_{\text{ISM}}$$

# Quantitative Comparison

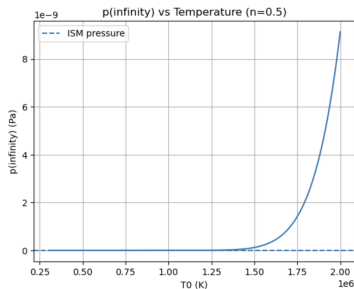
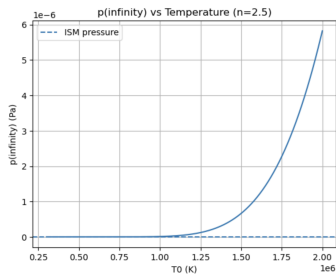
$$\frac{p(\infty)}{p_{\text{ISM}}}$$

- $n = 2.5$ :  $\sim 5 \times 10^7$
- $n = 0.5$ :  $\sim 4 \times 10^4$

## Conclusion:

- Enormous pressure mismatch

# Dependence on Coronal Temperature



## Observation:

- $p(\infty)$  increases rapidly with  $T_0$

Hotter corona  $\Rightarrow$  stronger inconsistency

# Critical Value of $n$

$$n_{\text{crit}} \approx 0.27$$

## Interpretation:

- Only for  $n < 0.27$ :

$$p(\infty) \sim p_{\text{ISM}}$$

## But physically:

- Ionized plasma:  $n \approx 2.5$
- Neutral gas:  $n \approx 0.5$

$$\Rightarrow n \gg n_{\text{crit}}$$

# Final Conclusion (Numerical-1)

$$p(\infty) \gg p_{\text{ISM}}$$

⇒ Pressure mismatch

⇒ No hydrostatic equilibrium

Solar corona must expand outward

# Generalized Temperature Model

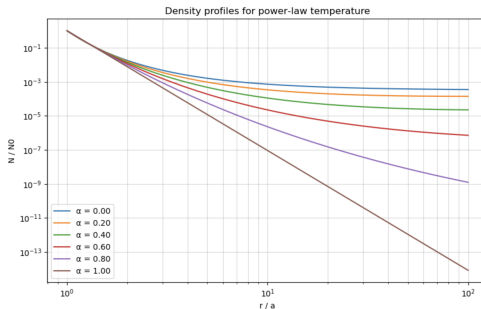
$$T(r) = T_0 \left( \frac{a}{r} \right)^\alpha$$

$$\frac{dN}{dr} = -\frac{GM_\odot m_p}{2k_B T r^2} N - \frac{N}{T} \frac{dT}{dr}$$

- Extends Parker's static equilibrium to arbitrary temperature gradients
- Includes isothermal ( $\alpha = 0$ ) and conductive case ( $\alpha = 2/7$ )

**Implementation:** GitHub) Repository

# Density Profiles for Power-Law Temperature



- Increasing  $\alpha \rightarrow$  faster temperature decay
- Stronger density drop at large radii

Higher  $\alpha \Rightarrow$  weaker outer corona

# Asymptotic Pressure vs Temperature Gradient

| $\alpha$ | $p_{\infty}$ (dyne/cm <sup>2</sup> ) | Comparison with ISM   |
|----------|--------------------------------------|-----------------------|
| 0.00     | $2.90 \times 10^{-6}$                | $\gg p_{\text{ISM}}$  |
| 0.20     | $4.65 \times 10^{-7}$                | $\gg p_{\text{ISM}}$  |
| 0.40     | $3.00 \times 10^{-8}$                | $\gg p_{\text{ISM}}$  |
| 0.60     | $3.99 \times 10^{-10}$               | $\gg p_{\text{ISM}}$  |
| 0.80     | $3.04 \times 10^{-13}$               | $\sim p_{\text{ISM}}$ |
| 1.00     | $1.07 \times 10^{-18}$               | $\ll p_{\text{ISM}}$  |

$$p_{\text{ISM}} = 1.4 \times 10^{-13} \text{ dyne/cm}^2$$

# Physical Interpretation

- Pressure at infinity decreases as  $\alpha$  increases
- Confinement requires:

$$\alpha \gtrsim 0.8$$

- But conductive corona gives:

$$\alpha = \frac{2}{7} \approx 0.286$$

$$\Rightarrow p_{\infty} \sim 5 \times 10^{-7} \gg p_{\text{ISM}}$$

# Critical Temperature for Confinement

$$T_0^{\text{crit}} \approx 4.5 \times 10^5 \text{ K}$$

- Required for:

$$p_\infty = p_{\text{ISM}}$$

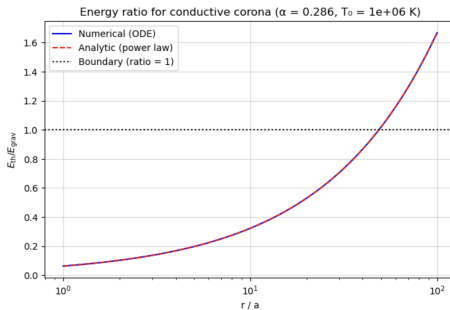
**But observed:**

$$T_0 \sim 10^6 \text{ K}$$

$$\Rightarrow T_0 \gg T_0^{\text{crit}}$$

Static corona inconsistent with observations

# Energy Ratio: Bound vs Unbound Gas



$$\epsilon(r) = \frac{E_{th}}{E_{grav}}$$

$$\epsilon(100a) \approx 1.67 > 1$$

$\Rightarrow$  Gas becomes unbound

# Comparison with Analytical Expectation

- Analytical:

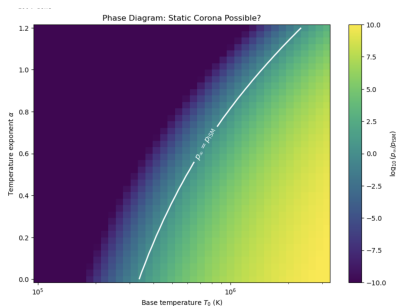
$$\epsilon(r) \propto r^{1-\alpha}$$

- For  $\alpha < 1$ :

$$\epsilon(r) \uparrow \text{ with } r$$

Escape inevitable at large distances

# Phase Diagram: Static Corona Possible?



- White curve:  $p_\infty = p_{ISM}$
- Above curve  $\rightarrow$  confined
- Below curve  $\rightarrow$  unconfined

# Where Does the Real Corona Lie?

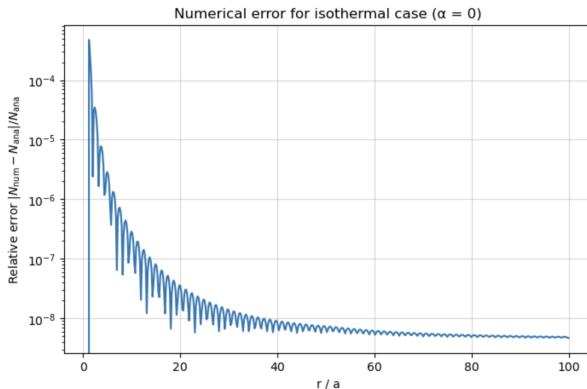
- Observed:

$$T_0 \sim 10^6 \text{ K}, \quad \alpha \sim 0.286$$

$\Rightarrow$  Deep in unconfined region

Static equilibrium impossible in real parameter space

# Numerical Validation



Max error  $\approx 4.8 \times 10^{-4}$

Mean error  $\approx 2.3 \times 10^{-6}$

High numerical accuracy

# Final Conclusion

- Realistic corona:

$$T_0 \sim 10^6 \text{ K}, \quad \alpha \sim 0.3$$

- Results:

$$p_\infty \gg p_{\text{ISM}}$$

- Energy:

$$E_{\text{th}} > E_{\text{grav}}$$

Static corona cannot exist

Solar wind is inevitable

## Sattwik

## Stationary Expansion

In the last section, we have shown that there is no hydrostatic equilibrium solution with **vanishing pressure at infinity**. We shall now consider what **steady expansion** we might expect from the solar corona.

We do not quantitatively know the distribution of heat in the solar corona over  $r$  and to what distance it extends. Hence, we merely assume that  $T(r)$  is given.

Now, the **Euler equation** for the stationary expansion of the solar corona reads,

$$\begin{aligned}\rho v \frac{dv}{dr} &= -\frac{dp}{dr} - \frac{GM_0}{r^2} \rho \\ NMv \frac{dv}{dr} &= -\frac{d(2Nk_B T)}{dr} - \frac{GNMM_0}{r^2}\end{aligned}\tag{1}$$

Similarly, from the **equation of continuity**,

$$\begin{aligned}\frac{d}{dr}(r^2 \rho v) &= 0 \\ \Rightarrow \frac{d}{dr}(r^2 N v) &= 0\end{aligned}\tag{2}$$

Imposing the condition that at  $r = a$ ;  $N = N_0$  and  $v = v_0$ , we find,

$$N(r)v(r) = N_0 v_0 \left(\frac{a}{r}\right)^2\tag{3}$$

Define **dimensionless** variables as follows,

$$\xi = \frac{r}{a}; \tau = \frac{T(r)}{T_0}; \lambda = \frac{GMM_0}{2k_B T_0 a}; \psi = \frac{\frac{1}{2}mv^2}{k_B T_0}$$

Using equation (3) to eliminate  $N(r)$ , equation (1) may be reduced to,

$$\left[1 - \frac{\tau}{\psi}\right] \frac{d\psi}{d\xi} = -2\xi^2 \frac{d}{d\xi} \left(\frac{\tau}{\xi^2}\right) - \frac{2\lambda}{\xi^2}\tag{4}$$

From equation (4), we want  $\psi$  as a function of  $\xi$ .

- Let us suppose that the temperature is maintained (by heating mechanisms) at the uniform value  $T_0$  from  $r = a$  to  $r = b$ .
- Beyond  $r = b$ , we suppose that the heating vanishes. The reason for this assumption is that, since the outward expansion of the corona consumes  $1.5 \times 10^{29}$  erg/sec of energy and thermal conduction can't transport even 1% as much, we may take  $T$  to be negligible beyond  $r = b$ .

Then, for  $a < r < b$ , we have,  $\tau = 1$  and equation (4) becomes,

$$\left[1 - \frac{1}{\psi}\right] \frac{d\psi}{d\xi} = -2\xi^2 \frac{d}{d\xi} \left(\frac{1}{\xi^2}\right) - \frac{2\lambda}{\xi^2}$$

Imposing the condition  $\psi = \psi_0$  at  $r = a$ , we get,

$$\boxed{\psi - \ln\psi = \psi_0 - \ln\psi_0 + 4\ln\xi - 2\lambda\left(1 - \frac{1}{\xi}\right)} \quad (5)$$

Beyond  $r = b$ , we have,  $\tau \approx 0$ . Then,

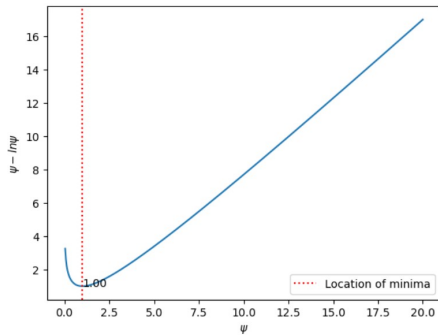
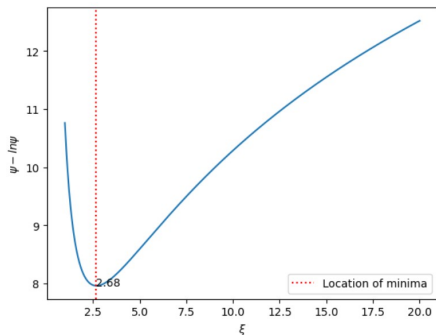
$$\begin{aligned}\frac{d\psi}{d\xi} &= -\frac{2\lambda}{\xi^2} \\ \implies \psi(\xi) &= \psi(b/a) - 2\lambda \left[ \frac{a}{b} - \frac{1}{\xi} \right]\end{aligned}\tag{6}$$

Now, steady outward flow requires a definite value of  $v_0$  for any given  $T_0$ . This is ensured by the fact that, equation (5) does not yield real values of  $v(r)$  for all  $r > a$  unless  $v_0$  has a particular value.

Rewriting equation (5),

$$\psi - \ln\psi = C_1 + 4\ln\xi + \frac{2\lambda}{\xi} \quad [C_1 = \psi_0 - \ln\psi_0 - 2\lambda]$$

An analysis of the behavior of  $\psi - \ln\psi$  as function of  $\xi$  and as a function of  $\psi$  gives us,



Given the fact that we must have a solution  $\psi$  which is **real** and **positive** for all values of  $\xi \geq 1$ , we may conclude,

$$\begin{aligned}\psi(\xi = \lambda/2) &= 1 \\ \psi_0 - \ln \psi_0 &= 2\lambda - 3 - 4\ln(\lambda/2)\end{aligned}\tag{7}$$

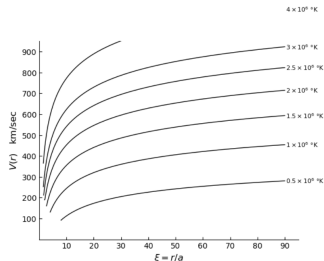
This equation gives a **definite value of  $v_0$  for any given  $T_0$** . Moreover, if we were, by some means, to pump gas out of the sun at a greater rate than the  $4\pi a^2 v_0 N_0$  given by equation (7), we would soon achieve a new steady outflow with the same  $v_0$  but with increased  $N_0$ , assuming  $T_0$  to remain unchanged.

Substituting equation (7) in equation (5), we get,

$$\psi - \ln \psi = -3 - 4 \ln(\lambda/2) + 4 \ln \xi + \frac{2\lambda}{\xi} \quad (8)$$

We then plot the outward gas velocity  $v(r)$  as a function of  $\xi = r/a$  for various temperatures  $T_0$ .

We take,  $a = 10^{11}$  cm and  $M = 1.66 \times 10^{-24}$  gm, so that  $\lambda = 8.0 \times 10^6 / T_0$  and  $v^2 = 1.7 \times 10^{-2} T_0 \psi$  (km/sec)<sup>2</sup>.



**Figure:** Plot of  $v(r)$  VS  $\xi(= r/a)$

This graph shows us that escape doesn't require particles to already exceed escape velocity when we have a pressure-driven flow. Thus, even the 160 km/sec thermal velocities of  $10^6$  K are sufficient to push gas out of the solar gravitational field having escape velocity 500 km/sec.

We now try to establish that this effect is **a result of spherical rather than 1-D expansion**.

Consider the general equation of continuity in **n-dimension** (in analogy with that derived for  $n = 3$  dimensions:

$$N(r)v(r) = N_0 v_0 \left(\frac{a}{r}\right)^{n-1} \quad (9)$$

Then, we end up with,

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 + 2(n-1)\ln\xi - 2\lambda\left(1 - \frac{1}{\xi}\right) \quad (10)$$

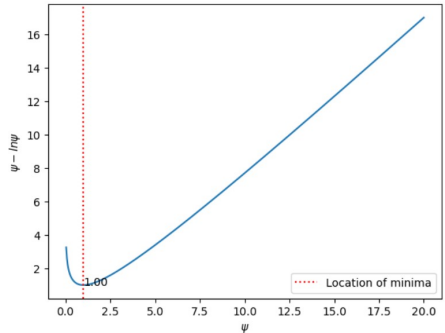
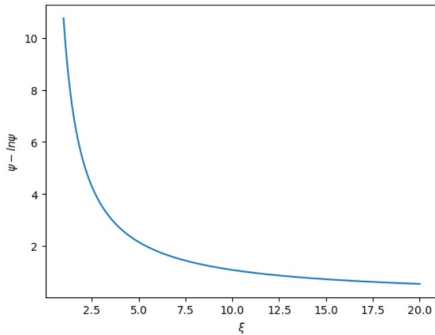
In equation (10), note that, the  $(\ln\xi)$ -term is solely responsible for  $v(r) \rightarrow \infty$  as  $r \rightarrow \infty$ .

For 1-D, this term actually vanishes and we have,

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 - 2\lambda\left(1 - \frac{1}{\xi}\right)$$

$$\implies \psi - \ln\psi = C_1 + \frac{2\lambda}{\xi} \quad [C_1 = \psi_0 - \ln\psi_0 - 2\lambda]$$

An analysis of the behavior of  $\psi - \ln\psi$  as function of  $\xi$  and as a function of  $\psi$  gives,



Note that,  $\psi - \ln\psi$  monotonically decreases in the domain  $\xi \in [1, \infty)$ .  
Hence, in order to have

a real (and positive) solution  $\psi$  for all values of  $\xi \geq 1$ , we must have,

$$\psi \leq 1 \quad [\text{for } 1 \leq \xi < \infty]$$
$$\implies v \leq \sqrt{\frac{2k_B T_0}{M}} \approx 129 \text{ km/sec}$$

Thus, for the 1-D expansion,  $v$  is limited to less than the thermal velocity, which is in contradiction to Biermann's observation of  $v \sim 500 - 1500 \text{ km/sec}$ . Hence, the **coronal expansion must be spherical** to obtain velocities of the order of  $\sim 500 \text{ km/sec}$  or more.

# The case of local spherical expansion

Suppose that, instead of spherical expansion centered about the sun, we consider a more local spherical expansion **centered around some hypothetical active region** on the surface of the sun and confined to a narrow cone, as shown in the following figure,

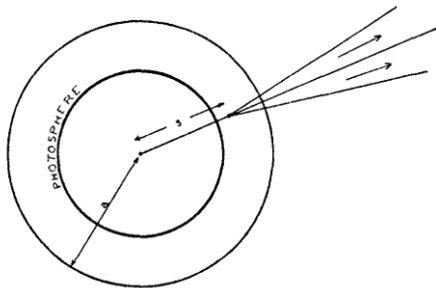


Figure: Local Spherical Expansion

In this case, the equation of continuity becomes,

$$N(r)v(r) = N_0 v_0 \left( \frac{a-s}{r-s} \right)^2 \quad (11)$$

where,  $s$  is the distance of the origin of spherical expansion from the centre of the sun.

Then, in place of equation (5), we obtain,

$$\psi - \ln \psi = \psi_0 - \ln \psi_0 + 4 \ln \left[ \left( \xi - \frac{s}{a} \right) / \left( 1 - \frac{s}{a} \right) \right] - 2\lambda \left( 1 - \frac{1}{\xi} \right) \quad (12)$$

We now write,

$$\psi - \ln \psi = C_1 + 4 \ln \left[ \left( \xi - \frac{s}{a} \right) / \left( 1 - \frac{s}{a} \right) \right] + \frac{2\lambda}{\xi}$$

and perform the same analysis as before.

Note that,  $\psi - \ln \psi \in [C_1 + 2\lambda, \infty)$  for  $\xi \in [1, \infty)$ .

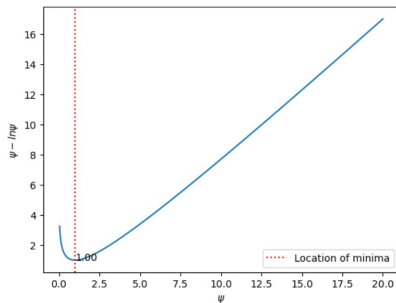
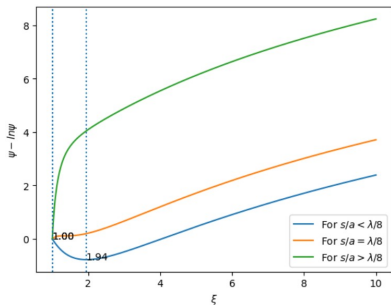
Let,  $f(\xi) = 4 \ln \left[ \left( \xi - \frac{s}{a} \right) / \left( 1 - \frac{s}{a} \right) \right] + \frac{2\lambda}{\xi}$ .

$f'(\xi) = 0$  gives,

$$\xi_{min} = \frac{\lambda + \sqrt{\lambda^2 - 8\lambda(s/a)}}{4} \quad (13)$$

For existence of  $\xi_{min} \in \mathbb{R}$ , we demand,

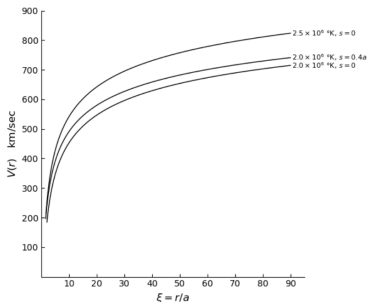
$$\frac{s}{a} < \frac{\lambda}{8}$$



In this case, we can find  $(\psi_0 - \ln \psi_0)$  and hence  $v_0$ .

For  $s/a \geq \lambda/8$ , the RHS of equation (12) increases monotonically and we cannot find a solution  $v_0$ .

We now plot  $v(r)$  as a function of  $\xi$ , taking the same values of constants as used above:



**Figure:** Plot of  $v(r)$  vs  $\xi(= r/a)$

We see that, when  $s > 0$ , larger outward velocities result from the same temperature.

# Summing up

- The graph of  $v(r)$  VS  $\xi$  (for spherical expansion **centered about the sun**) shows us that escape doesn't require particles to already exceed escape velocity when we have a **pressure-driven flow**.

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# Summing up

- The graph of  $v(r)$  VS  $\xi$  (for spherical expansion **centered about the sun**) shows us that escape doesn't require particles to already exceed escape velocity when we have a **pressure-driven flow**.
- The fact that **coronal expansion cannot be one dimensional** rules out a whole class of physical models and proves that a simple “tube-like” outward flow is inconsistent with a steady solar wind.
- Finally, the finding that when  $s > 0$ , larger outward velocities result from the same temperature shows that, the **final wind speed** is not set by temperature alone; it also **depends on how the flow expands** and thus explains why different regions on the Sun can produce **different solar-wind speeds** even **at similar temperatures**.

**Shivam**

## Coronal Heating and Mass Loss

# Energy Budget of the Expanding Corona

Using the value of  $N = 500 \text{ cm}^{-3}$  and  $v = 500 \text{ km/s}$  at Earth's orbit, we can calculate the rate of energy loss needed for expansion to be:

$$2\pi r^2 N M v^3$$

## Key Constraint

The expansion of the solar corona consumes at least  $1.5 \times 10^{29} \text{ erg s}^{-1}$ .

Two possible heating scenarios:

- 1 **Electromagnetic:** If  $4 \times 10^{-5}$  of escaping solar radiation ( $L_{\odot} = 3.8 \times 10^{33} \text{ erg s}^{-1}$ ) is absorbed by the corona, the energy supply is sufficient. However, due to the corona being optically thin, this is not physically plausible.

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- 2 **Mechanical:** If electromagnetic heating is ruled out, energy must be transported by acoustic or hydromagnetic waves from the photosphere outward.

# Kinetic Energy Flux Deficit

Suppose the corona is heated only up to  $r = b$ , where gas velocity reaches  $v_m$ .

Rate of energy transfer at  $r = b$ :

$$MN_m v_m^3 2\pi b^2 \text{ erg s}^{-1}$$

Rate at intermediate radius  $a < r < b$ :

$$MN(r) v(r)^3 2\pi r^2 \text{ erg s}^{-1}$$

Heating flux required at radius  $r$ :

$$I(r) = \frac{1}{2} M \left[ N_m v_m^3 \frac{b^2}{r^2} - N v^3 \right] \text{ erg cm}^{-2} \text{ s}^{-1}$$

Using continuity  $N(r)v(r) = N_0 v_0 \left(\frac{a}{r}\right)^2$ , this becomes:

$$I(r) = N_0 v_0 \left(\frac{a}{r}\right)^2 k T_0 (\psi_m - \psi) \text{ erg cm}^{-2} \text{ s}^{-1}$$

# Defining the Transport Velocity $w$

Let  $w$  be the effective velocity at which mechanical waves move, transporting energy density  $2NkT$ . Let  $v$  be the bulk outflow velocity. Net energy flux:

$$I(r) = 2wNkT_0 + 2vNkT_0$$

## Effective Transport Velocity

Equating expressions for  $I(r)$ :

$$w = v \left[ \frac{1}{2}(\psi_m - \psi) - 1 \right]$$

# Physical Constraint on $w/u$

The ratio with thermal velocity  $u = (3kT_0/M)^{1/2}$ :

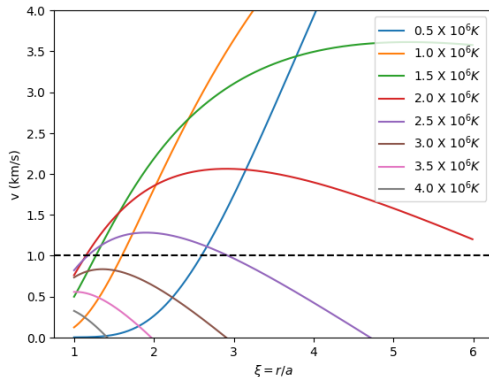
$$\frac{w}{u} = \left(\frac{2}{3}\right)^{1/2} \psi^{1/2} \left[ \frac{1}{2}(\psi_m - \psi) - 1 \right]$$

## Self-Consistency Requirement

$$\frac{w}{u} \lesssim 1$$

Mechanical wave speed must not exceed thermal speed significantly.

# $w/u$ vs $\xi$ plot at different temperatures



- $\psi_m$  corresponds to  $v = 500 \text{ km/s}$
- Physically plausible temperatures are above  $3.0 \times 10^6 \text{ K}$

# Rate of Solar Mass Loss

Using  $N = 500 \text{ cm}^{-3}$  and  $v = 500 \text{ km/s}$ , Biermann estimated mass loss rate  $\sim 10^{14} \text{ g s}^{-1}$ .

Mass Loss Formula at  $r = a$

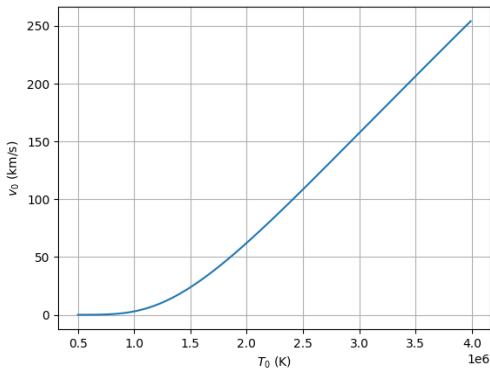
$$\frac{dM_{\odot}}{dt} = 4\pi a^2 N_0 M v_0$$

with  $a = 10^{11} \text{ cm}$ ,  $N_0 = 3 \times 10^7 \text{ cm}^{-3}$ .

Required:

$$v_0 \simeq 160 \text{ km s}^{-1}$$

# $v_0$ vs Temperature

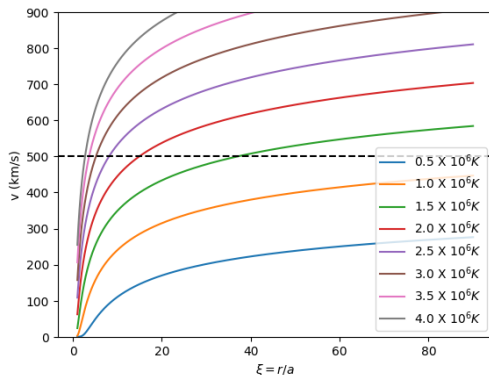


- $v_0$  obtained from:

$$\psi_0 - \ln(\psi_0) = 2\lambda - 3 - 4 \ln\left(\frac{\lambda}{2}\right)$$

- Required velocity achieved at  $T_0 = 3.0 \times 10^6$  K

# Verification of Terminal Velocity



- At  $T_0 = 3.0 \times 10^6 \text{ K}$ , terminal velocity  $\approx 500 \text{ km/s}$  is achieved.

# Self-Consistent Model

All Conditions Satisfied for  $T_0 = 3 \times 10^6 \text{ K}$

|                      |                                       |
|----------------------|---------------------------------------|
| Mass loss rate       | $10^{14} \text{ g s}^{-1} \checkmark$ |
| Terminal velocity    | $500 \text{ km s}^{-1} \checkmark$    |
| Transport constraint | $w \leq u \checkmark$                 |

## Conclusion

A coronal temperature of  $2\text{--}3 \times 10^6 \text{ K}$  provides a self-consistent physical explanation of solar wind outflow.

# Model Assumptions and Limitations

## Assumptions

- $a = 10^6$  km chosen as effective coronal base
- Spherical symmetry assumed
- Uniform temperature  $T_0$  between  $r = a$  and  $r = b$

## Limitation

Observational constraints do not allow a more detailed radial temperature model  $T(r)$ .

# Susnata

# General Solar Magnetic Field

- Till now we have ignored the presence of magnetic field.

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- Now we consider the effect of general outflow of solar gas, upon a general solar **dipole** field.
- **Question:** Does the solar wind ( $\mathbf{v}$ ) carry the Sun's magnetic field ( $\mathbf{B}$ ) with it, or does it move independently of it?

# Field Free Outflow

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- Then, plasma flows up **between** the lines of force.
- Minimal interaction with global solar magnetic field → thus field has no such effect on the wind.
- Only expect a few local perturbations in the field which is very nearly an **ordinary magnetic dipole**.

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- As a result, magnetic field lines are forced to **move with the plasma**.

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We conclude,

The gas flowing out from the Sun is **NOT field free**, and it is **threaded by magnetic lines of force** and carries them outward.

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in the frame of reference co-rotating with the Sun.

$$\nabla \cdot \mathbf{B} = 0 \implies \alpha = \alpha(\theta, \phi_0) \left( \frac{b}{r} \right)^2$$

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along a given streamline (with azimuth  $\phi_0$  at  $r = b$ )

# Parker (1958) Model

- In the **inertial frame** the **gas velocity**  $\forall r \geq b$ , is

$$v_r = v_m,$$

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- So, in the frame of reference rotating with the sun we have the gas velocity at any point  $r \geq b$

$$v_r = v_m,$$

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- it is obvious that the lines of force of the magnetic field coincide with the velocity **streamlines**.

## Derivation

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- Separately, setting each component to zero, we get 3 ODEs

$$\boxed{\frac{dr}{v_r} = \frac{r d\theta}{v_\theta} = \frac{r \sin \theta d\phi}{v_\phi}}$$

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Using

$$\Omega_{\odot} \approx 2.7 \times 10^{-6} \text{ rad/s} ; v_m = 1000 \text{ km/s} ; b = 5 \times 10^6 \text{ km}$$

We plot the projection of the lines of force of any solar field, on the **solar equatorial plane** ( $\theta = \pi/2$ ).

# Parker Spiral

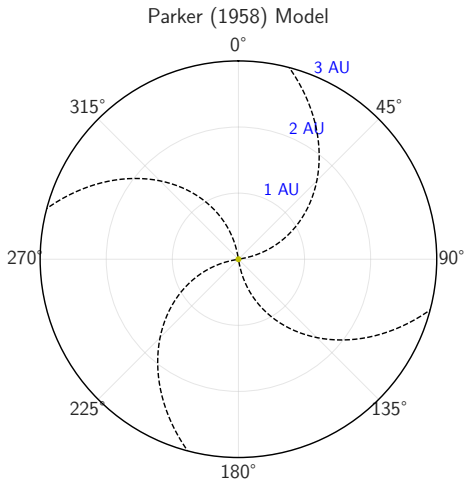


Figure: Parker Spiral projection in the plane  $\theta = \pi/2$

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$$\begin{aligned} \frac{d\mathbf{L}}{dr} &= -\frac{d(v_\phi r)}{dr} \hat{\theta} = -\left[ \frac{\partial(v_\phi r)}{\partial r} + \frac{\partial(v_\phi r)}{\partial \theta} \frac{d\theta}{dr} \right] \hat{\theta} \\ &= -\Omega_\odot b \sin \theta \hat{\theta} \neq 0 ! \end{aligned}$$

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Thus angular momentum is NOT conserved locally, as the gas moves outward.

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- Parker assumed this to be a **constant**.
- Also note that, for electric fields to remain continuous across the inner boundary ( $r = b$ ), we require azimuthal flow speed to also be **continuous**.

$$\lim_{\epsilon \rightarrow 0^+} v_\phi(b + \epsilon) = v_\phi(b)$$

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- $$[(\mathbf{v} \cdot \nabla) \mathbf{v}]_\phi = 0 \implies \frac{\partial v_\phi}{\partial r} = -\frac{v_\phi}{r} \implies \boxed{\frac{\partial(v_\phi r)}{\partial r} = 0}$$

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- $$[(\mathbf{v} \cdot \nabla) \mathbf{v}]_\phi = 0 \implies \frac{\partial v_\phi}{\partial r} = -\frac{v_\phi}{r} \implies \boxed{\frac{\partial(v_\phi r)}{\partial r} = 0}$$

- Clearly,  $v_\phi = \text{constant}$  is **NOT** a solution to this. **Thus, Parker's assumption does not satisfy conservation of angular momentum.**



$$\begin{aligned}\frac{\partial(v_\phi r)}{\partial r} &= 0 \\ \Rightarrow v_\phi(r)r &= v_\phi(b)b \\ \Rightarrow v_\phi &= \frac{\Omega_\odot b^2 \sin \theta}{r} !!\end{aligned}$$



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- So in **inertial frame** , gas velocity is of the form ( $\forall r > b$ ).

$$v_r = v_m,$$

$$v_\theta = 0,$$

$$v_\phi = \frac{\Omega_\odot b^2 \sin \theta}{r}$$

# Angular Momentum

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However, we must keep in mind that we have not considered the effect of B-field. Moreover, the global angular momentum of the entire plasma wind can not be conserved, and it leads to a phenomenon called **radiation braking** in Sun.

# Calculating **B**-field

- Similarly, in the frame of reference **co-rotating** with the Sun,

$$v_r = v_m,$$

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$$v_\phi = \frac{\Omega_\odot b^2 \sin \theta}{r} - \Omega_\odot r \sin \theta = \frac{\Omega_\odot b^2 \sin \theta}{r} \left[ 1 - \left( \frac{r}{b} \right)^2 \right]$$

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$$\mathbf{B} = \alpha \mathbf{v}$$

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- Recall,

$$\mathbf{B} = \alpha \mathbf{v}$$

- Doing exactly the same steps as before we get,

$$B_r = B(\theta, \phi_0) \left( \frac{b}{r} \right)^2,$$

$$B_\theta = 0,$$

$$B_\phi = -\frac{B(\theta, \phi_0)}{v_m} \frac{\Omega_\odot b^2 \sin \theta}{r} \left[ 1 - \left( \frac{b}{r} \right)^2 \right]$$

# Streamline Equation

Since the solar wind and magnetic field follow the same streamline. We can also write,

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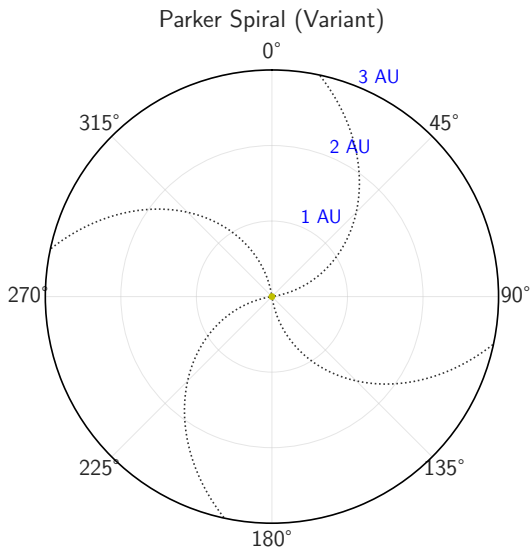
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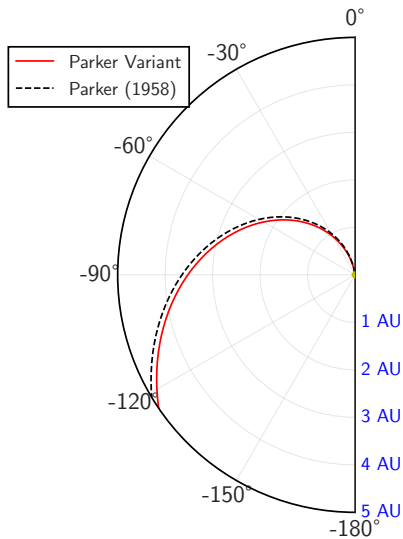
Plotting with the same value of parameters as before gives,

# Parker Spiral (Variant)



# Shape

We overlay the two spirals, and plot only in one quadrant



# Path Length

Numerically Calculate: **Difference in arc lengths** of the 2 form of spirals

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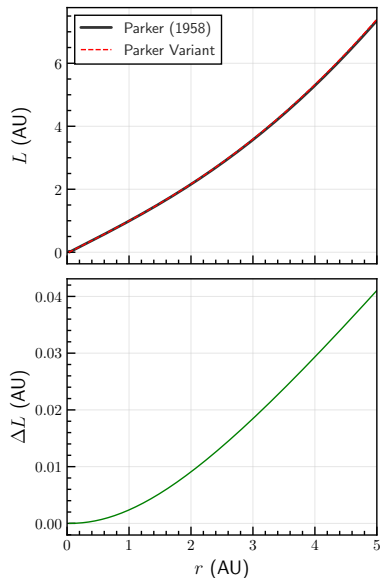
Substitute,

$$\phi_P(r) = -\frac{\Omega_{\odot}}{v_m} \left[ (r - b) - b \ln \frac{r}{b} \right] + \phi_0$$

$$\phi_2(r) = -\frac{\Omega_{\odot}}{v_m} \left[ (r - b) + b^2 \left( \frac{1}{r} - \frac{1}{b} \right) \right] + \phi_0$$

# Path Length

The difference in path length is fairly small.



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We determine the surface in **equatorial plane** ( $\theta = \pi/2$ ). Use the approximation,  $\frac{v_m}{b\Omega_\odot} \gg 1$

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$$B_r = B_\phi \implies R \simeq \frac{v_m}{\Omega_\odot} = 2.47 \text{ AU}$$

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- **Parker Variant Model**

$$B_r = B_\phi \implies R \simeq \frac{2b^2\Omega_\odot}{v_m} = 0.0009 \text{ AU}$$

$$B_r = B_\phi \text{ surface}$$

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- **Parker Variant Model**

$$B_r = B_\phi \implies R \simeq \frac{b^2 \Omega_\odot}{v_m} = 4.5 \times 10^{-4} \text{ AU}$$

Thus our variant, spirals much before.

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- Consistency can be verified with satellite observations

## Energy Densities: Order of Magnitude calculations

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$$u_B = \frac{(1 \text{ G})^2}{8\pi} = \frac{1}{8\pi} \approx \boxed{0.04 \text{ erg/cm}^3}$$

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  - Kinetic Energy Density:  $u_k \approx 0.01 \text{ erg/cm}^3$
- **Conclusion:** Near the solar corona, by order of magnitude calculation:

Magnetic Energy Density  $\approx$  Hydrostatic Pressure  
 $\approx$  Kinetic Energy Density

## Energy Density Fall-off Rate

# Magnetic Energy Density Fall-off rate

We define  $R = v_m \sin \theta / \Omega_\odot$  (the radius where  $B_r$  and  $B_\phi$  are equal)

- For  $r < R$ , the magnetic field behaves as:

$$B(r) \approx B_r \propto \frac{1}{r^2} \implies u_B(r) \propto \frac{1}{r^4}$$

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- In summary:

$$u_B(r) \propto \begin{cases} \frac{1}{r^4} & \text{for } r < R \\ \frac{1}{r^2} & \text{for } r > R \end{cases}$$

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- The kinetic energy density can be expressed as:

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- Since they are of the same order near the solar corona, we do not expect the magnetic energy density to dominate over the kinetic energy density at any distance.
- **Implication:** Therefore, we do not expect the general solar dipole field to significantly influence the motion of the outflowing gas, once the gas has left the solar corona.

# Solar Torque

# Maxwell Stress Tensor & Torque

- The radially outward flow of plasma and magnetic fields, plus the Sun's rotation, exerts a torque calculated via the Maxwell stress tensor:

$$T_{ij} = \frac{1}{4\pi} \left( B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

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- For a spherical surface of radius  $r$ , the torque magnitude about the z-axis is:

$$\tau_z = \int (\mathbf{r} \times \mathbf{T}_{r\phi}) dA = \frac{1}{4\pi} \int (r \sin \theta) (B_r B_\phi) dA$$

where  $dA = r^2 \sin \theta d\phi d\theta$ .

# Torque Integral & Asymptotic Limit

- The magnetic field components for radial outflow simplify the torque to:

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- The torque vanishes at  $r = b$  due to idealized velocity field adopted, completely ignoring magnetic stresses and approaches a constant as  $r \rightarrow \infty$ :

$$\tau_{\infty} = \frac{2}{15} b^4 \frac{\Omega_{\odot}}{v_m} B_0^2$$

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- **Conclusion:** The torque exerted on the sun by our interplanetary model of the solar field is not sufficient to cause significant deceleration of the sun's rotation over its lifetime.

# Plasma Instability

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- Within 1-2 AU, the solar wind expands primarily perpendicular to the radial magnetic field  $\mathbf{B}$ .
- In this tenuous plasma, the collision frequency is low; thus, energy is not redistributed among degrees of freedom.
- **Result:** Perpendicular pressure  $P_n$  becomes significantly lower than parallel pressure  $P_s$ .

# Alfven Waves in Anisotropic Plasma

- We start with Euler equation for a plasma with anisotropic pressure:

$$\rho \left( \frac{d\mathbf{u}_D}{dt} \right) = -\nabla \cdot \mathbf{P} + \frac{1}{c}(\mathbf{J} \times \mathbf{B})$$

where  $\mathbf{P} = P_n \mathbf{I} + (P_s - P_n) \hat{\mathbf{n}} \hat{\mathbf{n}}$  is the pressure tensor and  $\hat{\mathbf{n}}$  is the unit vector along the magnetic field.

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- Using this, we get the modified momentum equation for the electric drift velocity  $\mathbf{u}_D$ :

$$\rho \left( \frac{d\mathbf{u}_D}{dt} \right) = -\nabla_n (P_n + P_m) + \left[ \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi} \right]_n \left[ 1 + \frac{P_n - P_s}{2P_m} \right]$$

where  $P_m = B^2/8\pi$  is the magnetic pressure.

# Alfven Waves in Anisotropic Plasma

- We start with Euler equation for a plasma with anisotropic pressure:

$$\rho \left( \frac{d\mathbf{u}_D}{dt} \right) = -\nabla \cdot \mathbf{P} + \frac{1}{c}(\mathbf{J} \times \mathbf{B})$$

where  $\mathbf{P} = P_n \mathbf{I} + (P_s - P_n) \hat{\mathbf{n}} \hat{\mathbf{n}}$  is the pressure tensor and  $\hat{\mathbf{n}}$  is the unit vector along the magnetic field.

- Using this, we get the modified momentum equation for the electric drift velocity  $\mathbf{u}_D$ :

$$\rho \left( \frac{d\mathbf{u}_D}{dt} \right) = -\nabla_n (P_n + P_m) + \left[ \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi} \right]_n \left[ 1 + \frac{P_n - P_s}{2P_m} \right]$$

where  $P_m = B^2/8\pi$  is the magnetic pressure.

- We also have Alfven's frozen-in flux condition:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u}_D \times \mathbf{B})$$

# Linearized Momentum Equation

We consider small perturbations  $\delta u_x$  and  $\delta B_x$  in the Transverse direction (say  $x$ ), with the background magnetic field  $B_0$  and static velocity field  $u_0 = 0$  along the  $z$ -axis. Linearizing the momentum equation gives:

$$\rho \frac{\partial \delta u_x}{\partial t} = \frac{B_0}{4\pi} \frac{\partial \delta B_x}{\partial z} \left[ 1 + \frac{4\pi(P_n - P_s)}{B_0^2} \right]$$

Combined with the linearized induction equation (frozen-in flux):

$$\frac{\partial \delta B_x}{\partial t} = B_0 \frac{\partial \delta u_x}{\partial z}$$

# Wave Velocity and the Firehose Instability

Combining the previous equations yields the wave equation:

$$\frac{\partial^2 \delta B_x}{\partial t^2} = \left[ \frac{B_0^2}{4\pi\rho} + \frac{P_n - P_s}{\rho} \right] \frac{\partial^2 \delta B_x}{\partial z^2}$$

The propagation velocity  $v$  is therefore:

$$v = \frac{B_0}{\sqrt{4\pi\rho}} \left[ 1 + \frac{4\pi(P_n - P_s)}{B_0^2} \right]^{1/2}$$

**Stability Criterion:** If  $4\pi(P_s - P_n) > B^2$ , the velocity becomes purely imaginary, triggering the **Firehose Instability**.

# Physical Implications

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- We have seen above that for solar wind we can have  $P_s > P_n$  due to the expansion of the plasma.
- Due to the firehose instability, the wave doesn't propagate and it's amplitude instead grows exponentially, leading to turbulence in the plasma.
- It is estimated that a wave length of  $10^6$  km has a characteristic time of growth of the order of 4 hours; shorter wave lengths grow correspondingly faster.

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- Observational evidence of both the radial field inside 1 a.u. and the disorganized shell surrounding 1 a.u. comes from the onset and subsequent decay of cosmic rays from solar flares (Meyer, Parker, and Simpson 1956; Simpson 1957).
- Therefore, readers are advised against taking too literally any of the smooth idealized models which was constructed in this paper and further work is discussed in (Parker 1958 a, b).

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- We also discussed the *firehose instability* in the solar wind plasma and its implications for turbulence in the plasma.
- Finally, we noted that the idealized models of the solar magnetic field are complicated by a disorganized region extending from about 1 a.u. outward, and that observational evidence supports this conclusion.

**Some concluding remarks from this work**

# Conclusion I

- 1 First we calculated that, the solar corona must **expand** outward, and cannot be static.
- 2 Then, we show that escape doesn't require particles to already exceed escape velocity when we have a pressure-driven flow. Besides, we pointed out that 1-D coronal expansion is **inconsistent** with a steady solar wind and explained why different regions on the sun can produce different solar wind speeds even at similar temperatures.
- 3 The required energy for **coronal heating** can be supplied by mechanical waves at reasonable coronal temperature within the physical constraints. We have also verified that the observed **mass loss** and terminal velocity are both achievable at this temperature.
- 4 The gas flowing out from the Sun is **NOT field free**, and it is **threaded by magnetic lines of force** and carries them outward.
- 5 For Parker's (1958) model, solar rotation starts dominating the radial outflow of the solar wind beyond  $\sim 2.5$  AU.

## Conclusion II

- 6 We then calculated the torque exerted on the sun by our interplanetary model of the solar field and found that it is not sufficient to cause significant deceleration of the sun's rotation over its lifetime.
- 7 Finally, we noted that the idealized models of the solar magnetic field are complicated by a disorganized region extending from about 1 a.u. outward, and that observational evidence supports this conclusion.

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**THANK YOU SIR :)**

# References I

- 1 Parker, E.N., 1958. Dynamics of the interplanetary gas and magnetic fields. *Astrophysical Journal*, vol. 128, p. 664, 128, p.664.

# APPENDIX

**Sudipta**

# Backup: Hydrostatic Equation (Full Derivation)

Start from:

$$\frac{dp}{dr} = -\rho \frac{GM}{r^2}$$

Using:

$$\rho = Nm, \quad p = 2NkT$$

Thus:

$$\frac{d}{dr}(2NkT) = -Nm \frac{GM}{r^2}$$

Expand derivative:

$$\frac{d}{dr}(2NkT) = 2k \frac{d}{dr}(NT)$$

# Backup: Differential Equation for Density

Product rule:

$$\frac{d}{dr}(NT) = T \frac{dN}{dr} + N \frac{dT}{dr}$$

Thus:

$$2k \left( T \frac{dN}{dr} + N \frac{dT}{dr} \right) = -Nm \frac{GM}{r^2}$$

Divide by  $2kN$ :

$$\frac{T}{N} \frac{dN}{dr} + \frac{dT}{dr} = -\frac{GMm}{2kr^2}$$

Divide by  $T$ :

$$\frac{1}{N} \frac{dN}{dr} = -\frac{1}{T} \frac{dT}{dr} - \frac{GMm}{2kTr^2}$$

# Backup: Temperature Derivative

Thus:

$$\frac{d \ln N}{dr} = -\frac{1}{T} \frac{dT}{dr} - \frac{GMm}{2kTr^2}$$

Given:

$$T(r) = T_0 \left( \frac{a}{r} \right)^{1/(n+1)}$$

Take logarithm:

$$\ln T = \ln T_0 + \frac{1}{n+1} (\ln a - \ln r)$$

Differentiate:

$$\frac{1}{T} \frac{dT}{dr} = -\frac{1}{(n+1)r}$$

# Backup: Final ODE

Substitute:

$$\frac{d \ln N}{dr} = \frac{1}{(n+1)r} - \frac{GMm}{2kTr^2}$$

Insert temperature:

$$T = T_0 \left( \frac{a}{r} \right)^{1/(n+1)}$$

Now,

$$\ln \frac{N}{N_0} = \int_a^r \left[ \frac{1}{(n+1)r'} - \frac{\lambda}{r'} \left( \frac{r'}{a} \right)^{n/(n+1)} \right] dr'$$

# Backup: Integration

First term:

$$\int \frac{dr'}{(n+1)r'} = \frac{1}{n+1} \ln \frac{r}{a}$$

Second term substitution:

$$x = \left( \frac{r'}{a} \right)^{n/(n+1)}$$

Second integral:

$$I = \int \frac{\lambda}{r'} \left( \frac{r'}{a} \right)^{n/(n+1)} dr'$$

Define:

$$x = \left( \frac{r'}{a} \right)^{n/(n+1)}$$

# Backup: Integration (Change of Variable)

Take derivative:

$$\ln x = \frac{n}{n+1}(\ln r' - \ln a)$$

$$\frac{1}{x} \frac{dx}{dr'} = \frac{n}{n+1} \frac{1}{r'}$$

Thus:

$$\frac{dx}{dr'} = \frac{n}{n+1} \frac{x}{r'}$$

Rearrange:

$$\frac{1}{r'} x dr' = \frac{n+1}{n} dx$$

# Backup: Completing Integration

Thus:

$$\begin{aligned} I &= \lambda \int \frac{1}{r'} \left( \frac{r'}{a} \right)^{n/(n+1)} dr' \\ &= \lambda \int x \frac{dr'}{r'} \end{aligned}$$

Using substitution:

$$\begin{aligned} &= \lambda \int \frac{n+1}{n} dx \\ &= \lambda \frac{n+1}{n} x \end{aligned}$$

Back-substitute:

$$= \lambda \frac{n+1}{n} \left( \frac{r}{a} \right)^{n/(n+1)}$$

# Backup: Final Density Solution

Apply limits:

$$I = \lambda \frac{n+1}{n} \left[ \left( \frac{r}{a} \right)^{n/(n+1)} - 1 \right]$$

Combine both integrals:

$$\ln \frac{N}{N_0} = \frac{1}{n+1} \ln \frac{r}{a} - \lambda \frac{n+1}{n} \left[ \left( \frac{r}{a} \right)^{n/(n+1)} - 1 \right]$$

Exponentiate:

$$N(r) = N_0 \left( \frac{r}{a} \right)^{1/(n+1)} \exp \left[ -\lambda \frac{n+1}{n} \left( \left( \frac{r}{a} \right)^{n/(n+1)} - 1 \right) \right]$$

Rewriting:

$$N(r) = N_0 \left( \frac{a}{r} \right)^{1/(n+1)} \exp \left[ \frac{\lambda(n+1)}{n} \left( \left( \frac{a}{r} \right)^{n/(n+1)} - 1 \right) \right]$$

# Backup: Pressure Derivation

Equation of state:

$$p = 2NkT$$

Substitute:

$$N(r) = N_0 \left(\frac{a}{r}\right)^{1/(n+1)} \exp[\dots]$$

$$T(r) = T_0 \left(\frac{a}{r}\right)^{1/(n+1)}$$

Multiply:

$$p(r) = 2kN_0 T_0 \left(\frac{a}{r}\right)^{2/(n+1)} \exp[\dots]$$

Define:

$$p_0 = 2N_0 k T_0$$

# Backup: Asymptotic Behaviour

Thus:

$$p(r) = p_0 \left(\frac{a}{r}\right)^{2/(n+1)} \exp \left[ \frac{\lambda(n+1)}{n} \left( \left(\frac{a}{r}\right)^{n/(n+1)} - 1 \right) \right]$$

As:

$$r \rightarrow \infty$$

We have:

$$\left(\frac{a}{r}\right)^{n/(n+1)} \rightarrow 0$$

Thus exponential becomes:

$$\exp \left[ \frac{\lambda(n+1)}{n} (0 - 1) \right] = \exp \left[ -\frac{\lambda(n+1)}{n} \right]$$

# Backup: Dimensionless Parameter

Power-law term:

$$\left(\frac{a}{r}\right)^{2/(n+1)} \rightarrow 0$$

BUT density term compensates  $\rightarrow$  finite pressure

Final result:

$$p(\infty) = p_0 \exp \left[ -\frac{\lambda(n+1)}{n} \right]$$

Now,

$$\lambda = \frac{GMm}{2kT_0a}$$

Physical meaning:

- Ratio of gravitational energy to thermal energy

# Backup: Numerical Evaluation

Constants:

$$G = 6.67 \times 10^{-11}, \quad M = 2 \times 10^{30}$$

$$m = 1.67 \times 10^{-27}, \quad k = 1.38 \times 10^{-23}$$

$$T_0 = 1.5 \times 10^6, \quad a = 10^9$$

Compute numerator:

$$GMm = (6.67 \times 10^{-11})(2 \times 10^{30})(1.67 \times 10^{-27})$$

$$= 6.67 \times 2 \times 1.67 \times 10^{-8}$$

$$= 22.3 \times 10^{-8} = 2.23 \times 10^{-7}$$

## Backup: Lambda Calculation (cont.)

Denominator:

$$2kT_0a = 2(1.38 \times 10^{-23})(1.5 \times 10^6)(10^9)$$

$$= 2 \times 1.38 \times 1.5 \times 10^{-8}$$

$$= 4.14 \times 10^{-8}$$

Thus:

$$\lambda = \frac{2.23 \times 10^{-7}}{4.14 \times 10^{-8}}$$

$$\lambda \approx 5.35$$

## Backup: Pressure at Infinity

$$p(\infty) = p_0 \exp \left[ -\frac{\lambda(n+1)}{n} \right]$$

For example:

$$n = 2.5$$

$$\frac{n+1}{n} = \frac{3.5}{2.5} = 1.4$$

$$\lambda \frac{n+1}{n} = 5.35 \times 1.4 = 7.49$$

Thus:

$$p(\infty) = p_0 e^{-7.49}$$

$$e^{-7.49} \approx 5.5 \times 10^{-4}$$

# Backup: ISM Comparison

Base pressure:

$$p_0 = 2N_0 k T_0$$

$$N_0 \sim 3 \times 10^{13} \text{ m}^{-3}$$

$$p_0 \approx 10^{-3} \text{ Pa}$$

Thus:

$$p(\infty) \sim 10^{-7} \text{ Pa}$$

Interstellar medium:

$$p_{\text{ISM}} \sim 10^{-14} \text{ Pa}$$

$$\frac{p(\infty)}{p_{\text{ISM}}} \sim 10^7$$

# Backup: Logical Conclusion

$$p(\infty) \gg p_{\text{ISM}}$$

$\Rightarrow$  pressure mismatch

$\Rightarrow$  no equilibrium possible

$\Rightarrow$  static corona impossible

Corona must expand outward

# Susnata

**Solving for the scalar  $\alpha$**

# Solving for the scalar $\alpha$ I

To find the form of  $\alpha$ , we use the divergence-free condition of the magnetic field ( $\nabla \cdot \mathbf{B} = 0$ ) and the continuity equation for the plasma.

$$\nabla \cdot (\alpha \mathbf{v}) = 0 \implies \mathbf{v} \cdot \nabla \alpha + \alpha (\nabla \cdot \mathbf{v}) = 0$$

For steady flow with mass density  $\rho$ :

$$\nabla \cdot (\rho \mathbf{v}) = 0 \implies \mathbf{v} \cdot \nabla \rho + \rho (\nabla \cdot \mathbf{v}) = 0 \implies \nabla \cdot \mathbf{v} = -\frac{1}{\rho} (\mathbf{v} \cdot \nabla \rho)$$

Replace  $\nabla \cdot \mathbf{v}$  in the first equation,

$$\mathbf{v} \cdot \nabla \alpha - \frac{\alpha}{\rho} (\mathbf{v} \cdot \nabla \rho) = 0$$

Dividing by  $\rho$  yields,

$$\mathbf{v} \cdot \nabla \left( \frac{\alpha}{\rho} \right) = 0$$

# Solving for the scalar $\alpha$ II

This result shows that the ratio  $\alpha/\rho$  is **constant along any given streamline**. Since, we look at steady state, we have a constant mass loss rate  $\dot{M}_{\odot}$  from the sun, then in a spherically symmetric expansion we have,

$$\dot{M}_{\odot} = \rho(r) \times 4\pi r^2 v_r$$

As per our simple model, we assume  $v_r = v_m$  (a constant) for  $r > b$ . Therefore

$$\rho = \frac{\dot{M}_{\odot}}{4\pi r^2 v_m} \sim \frac{1}{r^2} \sim \alpha$$

Thus, along a given streamline (with  $\phi_0$  at  $r = b$ ) we have,

$$\alpha = \alpha(\theta, \phi_0) \left( \frac{b}{r} \right)^2$$

## Calculating $\vec{B}$ in Parker(1958) model

# Calculating $\vec{B}$ in Parker(1958) model I

## 1 Radial:

$$\begin{aligned} B_r &= \alpha v_m \\ &= \alpha(\theta, \phi_0) v_m \left( \frac{b}{r} \right)^2 \\ &\equiv B(\theta, \phi_0) \left( \frac{b}{r} \right)^2 \end{aligned}$$

## 2 Polar:

$$B_\theta = 0$$

# Calculating $\vec{B}$ in Parker(1958) model II

## 3 Azimuthal:

$$\begin{aligned} B_\phi &= \alpha(\theta, \phi_0) \left( \frac{b}{r} \right)^2 v_\phi \\ &= -B(\theta, \phi_0) \frac{\Omega_\odot (r - b)}{v_m} \left( \frac{b}{r} \right)^2 \sin \theta \end{aligned}$$

where  $\phi$ ,  $\phi_0$  are related by the [Streamline equation](#)  $[\phi_1(r)]$ . If we believe that only the solar dipole field threads the escaping gas, then

$$B(\theta, \phi_0) = B_0 \cos \theta$$