PH4201: Advanced Optics Laboratory

Spatial Light Modulator (SLM) Polarimetry

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Abstract

In this experiment we aim to determine the linear retardance (δ) and optical rotation (φ) of a Spatial Light Modulator (SLM) at different grey-scale values by measuring various Stokes vector elements.

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1 Theory

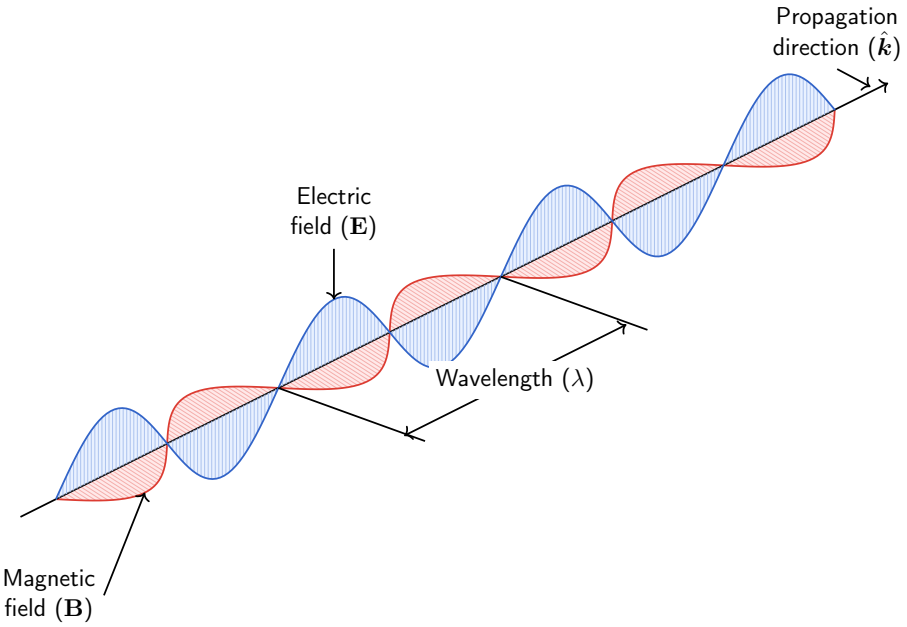


Figure 1: Cross-section of an EM wave propagating in the $\hat{k}$ direction, with **E** and the **B** fields oscillating in the mutually orthogonal planes.

Let a plane monochromatic wave propagate in the positive z direction. Hence the **E** and the **B** fields will oscillate in the xy plane, while the propagation vector $\hat{k}$ will be directed along z (see Fig. 1). Since the triplet of vectors **E**, **B** and $\hat{k}$ are mutually orthogonal, it suffices to look at only the behavior of vector **E**. The wave may be considered as the superposition of two waves with **E** having components along, say, the x and y directions, propagating along $\hat{k}$ -

$$E_x(z,t) = E_{0x}\,\exp[i(kz-\omega t)] = a_x \exp[-i\phi_x]\exp[i(kz-\omega t)]\,, \tag{1}$$
$$E_y(z,t) = E_{0y}\,\exp[i(kz-\omega t)] = a_y \exp[-i\phi_y]\exp[i(kz-\omega t)]\,. \tag{2}$$

The ratio of the complex amplitudes can be written as

$$\frac{E_{0y}}{E_{0x}} = \frac{a_y}{a_x}\exp[-i\Delta\phi], \tag{3}$$

where $\Delta\phi = \phi_y - \phi_x$ is the phase difference between the orthogonal components. Depending on the value of $\Delta\phi$, different polarization states are realized.

Polarization is a property which arises out of the transverse nature of the electromagnetic (EM) radiation, and is related to the orientation of the plane of vibration of its electric field. A linear polarization state can either be $E_x(z,t)$, $E_y(z,t)$ or a vector sum of these with $\phi_x = \phi_y$. For general elliptical polarization, $|E_{0x}| \neq |E_{0y}|$ and ($\phi_x - \phi_y =$ any arbitrary phase angle). The circular polarization state is a special case, where $|E_{0x}| = E_{0y}$ and ($\phi_x - \phi_y = \pi/2$).

Aside The wave propagation vector, can be written as $\hat{k} = \frac{2\pi}{\lambda}\tilde{n}$ where $\tilde{n} \in \mathbb{C}$. It can be shown that the imaginary part of the refractive index damps the electric (and magnetic) field as we move along $\vec{k}$, while the real part of refractive index gives a phase lag. We can roughly say that imaginary part of refractive index is related to attenuation or loss. While, the phase acquired due to the real part of refractive index is $\Delta\phi = (2\pi/\lambda)z \times \mathrm{Re}\{(\tilde{n})\}$.

1.1 Spatial Light Modulator (SLM)

The working principle of a Spatial Light Modulator (specifically the Liquid Crystal Display, or LCD type; Fig. 2) relies on using an electric field to *manipulate the orientation* of liquid crystals and consequently,

the phase of light.

When a monochromatic, linearly polarized laser beam is incident on an LCD-SLM, its phase is modulated by exploiting the electrically controlled birefringence of the liquid crystal layer. The SLM incorporates a pixelated layer of polarization-altering liquid crystals sandwiched between transparent polarizers.

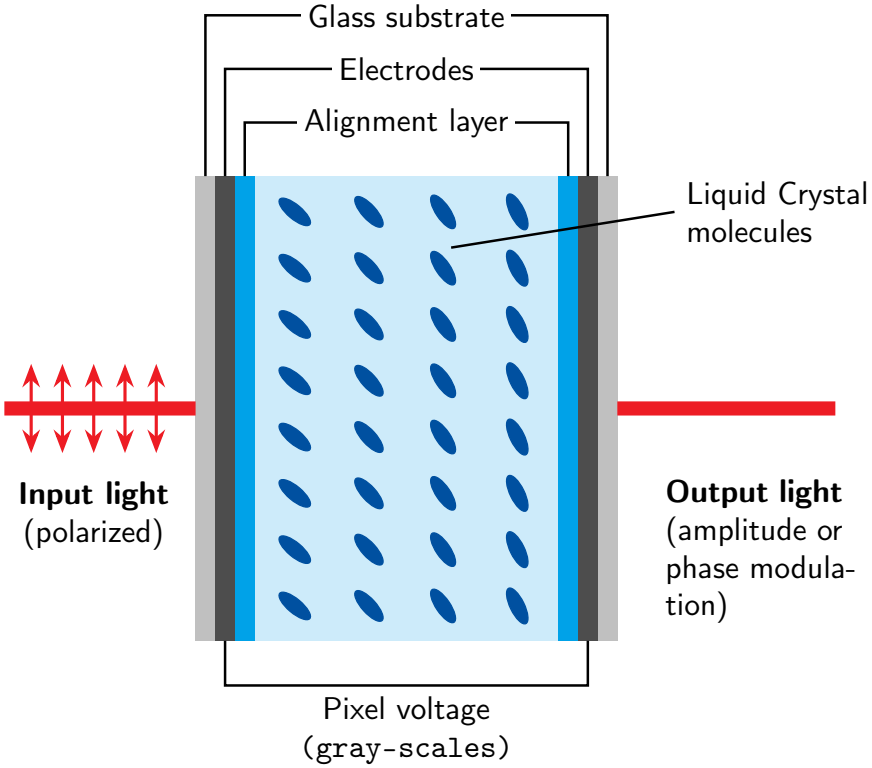


Figure 2: Schematic diagram of Liquid Crystal Display (LCD) spatial light modulator

When we *increase the gray level* (0 to 255), the device increases the voltage at that specific pixel, creating an electric field that forces the liquid crystal molecules above it to tilt or rotate. This physical reorientation *changes the effective refractive index* experienced by the light due to the birefringence of the crystals, thereby slowing the light down by a precise amount and inducing a specific phase shift relative to other parts of the beam. By feeding a computer-generated grayscale image—where the brightness of each pixel corresponds to a target phase delay—into the SLM software, we create a *phase mask* that spatially reshapes the wavefront of the transmitted laser, allowing us to generate complex interference patterns, focus the beam, or project holograms. Ultimately, the beam is transmitted out of the device with a spatially varying phase profile corresponding to the applied voltage pattern, all while keeping the light's amplitude effectively constant.

1.2 Characterising Polarisation States

It is clear that the amplitudes a_x and a_y in Eq. 3 cannot be measured directly in any experiment. Only intensities proportional to the squares of these amplitudes can be recorded by a detector. Thus, for characterizing the polarization states, Stokes proposed the parameters,

$$
\begin{aligned}
s_0 &= a_x^2 + a_y^2 \\
s_1 &= a_x^2 - a_y^2 \\
s_2 &= 2a_x a_y \cos\Delta\phi \\
s_3 &= 2a_x a_y \sin\Delta\phi
\end{aligned}
$$

where $\Delta\phi = \phi_x - \phi_y$ gives the phase difference between the orthogonal components. Only three Stokes parameters will be independent since only three independent quantities, namely, a_x, a_y and $\Delta\phi$,

are involved in describing them. Indeed, it is easy to verify that,

$$s_0^2 = s_1^2 + s_2^2 + s_3^2$$

Use of an analyzer makes it possible to measure the intensities of the two orthogonal polarizations a_x^2 and a_y^2. Thus s_0 is proportional to the intensity of light. Now let us see how to measure these various Stokes parameters.

1.2.1 Measurement

We know that in *reality* there is always some residual polarization due to reflection and scattering. Therefore, pure polarization states do not exist. Nor do the purely unpolarized states exist. It thus follows that Stokes parameters must be expressed in terms of mean intensities. In other words, Stokes parameters must be described in terms of *partially polarized light*.

Let the two orthogonal components be given by:

$$E_x^{(r)} = A_x \cos(\omega t + \phi_x)$$
$$E_y^{(r)} = A_y \cos(\omega t + \phi_y)$$

where A_x, ϕ_x, A_y and ϕ_y are slowly varying functions of time and the superscript r denotes the real field components. Defining the phase difference $\Delta\phi$ as $\Delta\phi = \phi_x - \phi_y$, the Stokes parameters can be written as:

$$s_0 = \langle A_x^2 \rangle + \langle A_y^2 \rangle$$
$$s_1 = \langle A_x^2 \rangle - \langle A_y^2 \rangle$$
$$s_2 = 2\langle A_x A_y \cos\Delta\phi \rangle$$
$$s_3 = 2\langle A_x A_y \sin\Delta\phi \rangle$$

The various stokes parameters can be measured in terms of various intensities. We use the complex notations,

$$E_x = A_x \exp\{[-i(\omega t + \phi_x)]\}$$
$$E_y = A_y \exp\{[-i(\omega t + \phi_y)]\}$$

Let a retardation plate producing retardation δ and a polarizer be put in the path of the beam. Let the principal direction of the polarizer be at an angle θ with the x-axis. The projection of the E-field on the principal direction can be written as:

$$E(\theta, \delta) = E_x \cos\theta + E_y e^{-i\delta} \sin\theta$$

The intensity of light described by the amplitude above can be expressed as:

$$\begin{aligned} I(\theta,\delta) &= \langle EE^* \rangle \\ &= \langle E_x E_x^* \rangle \cos^2\theta + \langle E_y E_y^* \rangle \sin^2\theta + (\langle E_x E_y^* \rangle e^{i\delta} + \langle E_y E_x^* \rangle e^{-i\delta}) \cos\theta \sin\theta \\ &\equiv I_x \cos^2\theta + I_y \sin^2\theta + [\langle A_x A_y \rangle e^{-i(\Delta\phi - \delta)} + \langle A_y A_x \rangle e^{+i(\Delta\phi - \delta)}] \cos\theta \sin\theta \end{aligned}$$

$$\therefore I = I_x \cos^2\theta + I_y \sin^2\theta + \langle A_x A_y \rangle \cos(\Delta\phi - \delta) \sin 2\theta \tag{4}$$

Here $I_{x,y} = \langle A^2_{x,y} \rangle$. Since,

$$
\begin{aligned}
s_0 &= I(0,0) + I(\pi/2,0) \equiv I_H + I_V \\
s_1 &= I(0,0) - I(\pi/2,0) \equiv I_H - I_V \\
s_2 &= I(\pi/4,0) - I(3\pi/4,0) \equiv I_P - I_M \\
s_3 &= I(\pi/4,\pi/2) - I(3\pi/4,\pi/2) \equiv I_R - I_L
\end{aligned}
$$

Hence is clear from Eq. 4 that a set of six measurements yielding $I(0,0)$, $I(\pi/2,0)$, $I(\pi/4,0)$, $I(3\pi/4,0)$, $I(\pi/4,\pi/2)$ and $I(3\pi/4,\pi/2)$ are adequate to determine the full set of Stokes parameters.

We note that, if any of the parameters s_1, s_2 or s_3 has a nonzero value, the light is polarized (at least partially) and $\sqrt{s_1^2 + s_2^2 + s_3^2}$ describes the intensity of the polarized portion of the beam, while the unpolarized portion is given by $s_0 - \sqrt{s_1^2 + s_2^2 + s_3^2}$. Thus we may define the following polarization parameters,

- **net degree of polarization**

$$P = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0}$$

- **degree of linear polarization**

$$P_L = \frac{\sqrt{s_1^2 + s_2^2}}{s_0}$$

- **degree of circular polarization**

$$P_C = \frac{s_3}{s_0}$$

1.2.2 Stokes' Vector Formalism

In this formalism, the state of polarization of a beam of light can be represented by four measurable quantities (intensities), when grouped in a 4×1 vector, are known as the Stokes vector, introduced by G. G. Stokes in 1852. They are defined as,

$$
\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \\ \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle \\ \langle E_x E_y^* \rangle + \langle E_y E_x^* \rangle \\ i(\langle E_y E_x^* \rangle - \langle E_x E_y^* \rangle) \end{bmatrix} = \begin{bmatrix} \langle E_{0x}^2 + E_{0y}^2 \rangle \\ \langle E_{0x}^2 - E_{0y}^2 \rangle \\ \langle 2E_{0x}E_{0y} \cos\Delta\phi \rangle \\ \langle 2E_{0x}E_{0y} \sin\Delta\phi \rangle \end{bmatrix}
$$

The notation $\langle \cdots \rangle$ denotes temporal averaging, and the electric field components E_{0x} and E_{0y}, along with the phase difference $\Delta\phi = \phi_y - \phi_x$, are also averaged over the measurement time.

The four Stokes parameters are can be further expressed relative to the following six intensity *photocurrent*[1] measurements (I) performed with ideal polarizers: I_H - horizontal linear polarizer (at 0 deg angle with respect to the horizontal axis of the laboratory frame); I_V - vertical linear polarizer (90 deg); I_P - 45 deg linear polarizer; I_M - 135 deg(- 45 deg) linear polarizer; I_R - right circular polarizer, and I_L - left circular polarizer. The Stokes vector ($\mathbf{S}$) is defined as

$$
\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_P - I_M \\ I_R - I_L \end{bmatrix} \tag{5}
$$

where S_0, S_1, S_2 and S_3 are Stokes vector elements. S_0 or I is the total detected light intensity which corresponds to addition of the two orthogonal component intensities, S_1 or Q is the difference in intensity between horizontal and vertical polarization states, S_2 or U is the portion of the intensity that corresponds to the difference between intensities of linear $+45°$ and $-45°$ ($135°$) polarization states, and S_3 or V is the difference between intensities of right circular and left circular polarization states.

The Stokes vectors corresponding to the different polarization states are represented as

[1] since photocurrent is directly proportional to the incident intensity of light

Unpolarized state	Horizontal polarization (0° or **H**)	Vertical polarization (90° or **V**)	+45° polarization (**P**)	135° (−45°) polarization (**M**)	Left circular polarization (LCP or **L**)	Right circular polarization (RCP or **R**)
$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$

1.3 Interaction of Polarised Light in medium

1.3.1 Mueller Matrices

So far we have discussed the polarization properties of a light beam. However, any optical element (or medium through which light propagates) may also possess intrinsic polarization characteristics. Thus the polarization properties of light may get altered (or destroyed even) upon interacting with any optical element. The interaction of polarized light with optical element is mathematically described by a 4×4 matrix $\mathbf{M}$, known as the **Mueller matrix** (named after its inventor Hans Mueller). The Mueller matrix describes the transfer function of any medium in its interaction with polarized light. The two descriptors are linked in the so-called *Stokes-Mueller calculus* – given a beam with a Stokes vector $\mathbf{S}_i$ incident on an optical element with a Mueller matrix $\mathbf{M}$, the light output from that element will have its polarization described by a Stokes vector as

$$\mathbf{S}_o = \mathbf{M}\mathbf{S}_i$$

The general form of a Mueller matrix of a linear retarder with magnitude of retardance δ and axis of birefringence oriented at an angle θ with respect to the horizontal (laboratory frame) is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta \cos\delta & \sin 2\theta \cos 2\theta(1-\cos\delta) & -\sin 2\theta \sin\delta \\ 0 & \sin 2\theta \cos 2\theta(1-\cos\delta) & \sin^2 2\theta + \cos^2 2\theta \cos\delta & \cos 2\theta \sin\delta \\ 0 & \sin 2\theta \sin\delta & -\cos 2\theta \sin\delta & \cos\delta \end{pmatrix} \tag{6}$$

Similarly, Mueller matrix for a circular retarder with retardance δ_C or optical rotation φ is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & -\sin 2\varphi & 0 \\ 0 & \sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

1.3.2 for SLM

An SLM is made of a stack of linearly birefringent materials of equal thickness, each rotated by the same amount relative to the previous. The resultant mueller matrix of such a configuration turns out to be a multiplication of retardance and rotation matrices.

$$\mathbf{M}_{\text{SLM}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta \cos\delta & \sin 2\theta \cos 2\theta(1-\cos\delta) & -\sin 2\theta \sin\delta \\ 0 & \sin 2\theta \cos 2\theta(1-\cos\delta) & \sin^2 2\theta + \cos^2 2\theta \cos\delta & \cos 2\theta \sin\delta \\ 0 & \sin 2\theta \sin\delta & -\cos 2\theta \sin\delta & \cos\delta \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & -\sin 2\varphi & 0 \\ 0 & \sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.3.3 Calculating Output Stokes Vectors

For incident horizontally (0°) linearly polarized light (input Stokes vector $\mathbf{S}_{\rightarrow} = [1\ 1\ 0\ 0]^T$), the Stokes vector ($\mathbf{S_H}$) of light emerging from the SLM is,

$$\mathbf{M}_{\text{SLM}}\mathbf{S}_{\rightarrow} = \mathbf{S_H}$$

$$\mathbf{S_H} = \begin{pmatrix} 1 \\ \cos 2\theta \cos(2\theta - 2\varphi) + \sin 2\theta \cos\delta \sin(2\theta - 2\varphi) \\ \sin 2\theta \cos(2\theta - 2\varphi) - \cos 2\theta \cos\delta \sin(2\theta - 2\varphi) \\ \sin(2\theta - 2\varphi) \sin\delta \end{pmatrix}$$

Similarly, for incident +45° linearly polarized light (input Stokes vector $\mathbf{S}_{\nearrow} = [1\ 0\ 1\ 0]^T$), the Stokes vector ($\mathbf{S_P}$) of emerging light emerging from the SLM is,

$$\mathbf{M}_{\text{SLM}}\mathbf{S}_{\nearrow} = \mathbf{S_P}$$

$$\mathbf{S_P} = \begin{pmatrix} 1 \\ -\cos\delta \sin 2\theta \cos(2\theta - 2\varphi) + \cos 2\theta \sin(2\theta - 2\varphi) \\ \cos\delta \cos 2\theta \cos(2\theta - 2\varphi) + \sin 2\theta \sin(2\theta - 2\varphi) \\ -\cos(2\theta - 2\varphi) \sin\delta \end{pmatrix}$$

2 Working Formula

Now, passing H and P(45°) polarized light through the SLM, and obtaining the output Stokes vectors $\mathbf{S_H}$, $\mathbf{S_P}$, we use the following relations to calculate the **linear retardance** (δ), and **optical rotation** (φ) as a function of grey-scale values.

$$\mathbf{S_H}(2) + \mathbf{S_P}(3) = (1 + \cos\delta)\cos 2\varphi \equiv X, \tag{8}$$

$$\mathbf{S_H}(3) - \mathbf{S_P}(2) = (1 + \cos\delta)\sin 2\varphi \equiv Y, \tag{9}$$

$$\tag{10}$$

$$\boxed{\delta = \cos^{-1}\left[\sqrt{X^2 + Y^2} - 1\right]} \tag{11}$$

$$\boxed{\varphi = \frac{1}{2}\tan^{-1}\left(\frac{Y}{X}\right)} \tag{12}$$

Here $\mathbf{S_{H/P}}(i)$ is the i^{th} component of the output Stokes vector $\mathbf{S_{H/P}}$. The 4 components of a Stokes vector are henceforth denoted by S_0, S_1, S_2 & S_3.

3 Experimental Setup

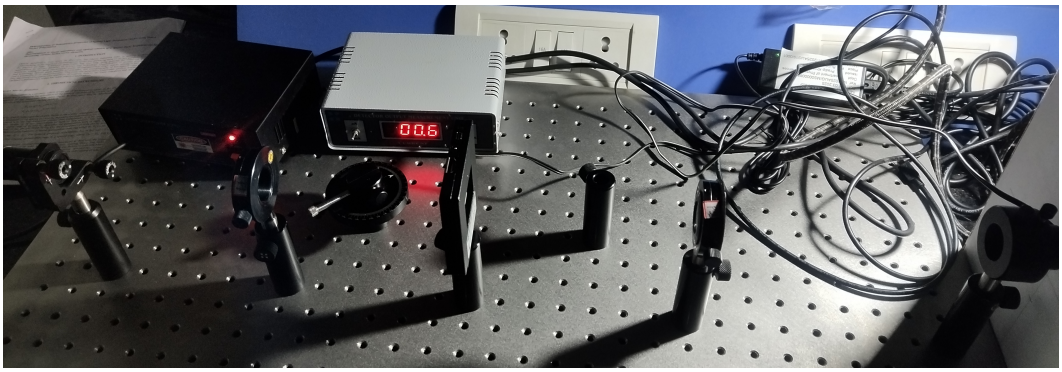


Figure 3: The setup at the laboratory

Our experimental setup (see Fig 3) consists of a monochromatic laser diode (655 nm), two linear polarizers (P_1 and P_2), one liquid crystal display spatial light modulator (LCD-SLM), one neutral density (ND) filter, and a photodetector setup. As shown in the above figure, the laser light is first passed through

a ND filter (was put after taking the picture)[2], then through the first polarizer, the SLM, and finally through the second polarizer before falling on the detector.

To begin with, the beam height was carefully aligned to be horizontal, along with also keeping in mind not to let the back-reflection fall directly in the laser source hole (this is bad for the laser). Then, the first polarizer's orientation was determined using the *brewster's angle method* (for glass plate it is $\theta_B = 56.3°$) and that of the second by measuring the intensity of the light passed through both.

We note that, determination of linear retardance δ and optical rotation ψ for pure linear and circularly birefringent samples respectively, do not involve circular polarization state measurements. So in this experiment, these are therefore determined by performing Stokes vector measurements involving linear polarization measurements alone (the first three elements of the Stokes vector are used, the fourth element involving circular polarization is excluded).

4 Data Analysis

4.1 Determination of Horizontal and Vertical axes of polarizers

Ideally, the polarization axis of a polarizer should be aligned with the zero-degree mark of the angular scale. However, in the present experimental setup this condition was not satisfied. It was therefore necessary to determine the actual orientation of the polarization axis. Once this reference was established, the P and M polarization axes could be identified solely from the angle corresponding to the horizontal polarization direction.

To determine this reference, a thin glass slab was used and unpolarized diode laser light of wavelength 655 nm was incident on the slab at the Brewster angle, $\theta_B = 56.3°$.

At the Brewster angle, the reflected light consists exclusively of S-polarized light, which is **vertically polarized**. This reflected beam was passed through the polarizer P_1, and the angular position at which the transmitted intensity was minimum was recorded. Hence, this angle was identified as the orientation corresponding to *horizontally* polarization angle (H) and is denoted by θ_H. The measured value is reported to be $\theta_H = 128 \pm 2°$

Consequently, the vertical polarization angle of polarizer P_1 is given by $\theta_H + 90°$. The orientations corresponding to the P and M polarization axes are therefore

$$P = \theta_H + 45°, \qquad M = \theta_H - 45°.$$

To determine the vertical polarization angle of the second polarizer P_2, placed between P_1 and detector. The angular position ξ corresponding to the minimum transmitted intensity was noted. This angle ξ was identified as the *vertical polarization*(V) direction of polarizer P_2, and is measured as $\xi = 34 \pm 2°$. Accordingly, the horizontal polarization angle (H) is given by $\xi + 90°$, while the P and M polarization axes for P_2 are

$$P = \xi - 45°, \qquad M = \xi + 45°.$$

4.2 Determination of δ and φ of the SLM

Table 1: Tabulation of Stokes vector measurements for different SLM grey-scale values

Input polarization states	Measured Photo-detector current in (μA)				Measured output Stokes vectors[Normalised]	X	Y	Linear retardance δ [deg]	Optical rotation φ [deg]
	I_H	I_V	I_P	I_M					
SLM at 0% Grey Scale									

[2] to reduce the intensity of light and prevent the saturation of the detector

Input polarization states	Measured Photo-detector current in (μA)				Measured output Stokes vectors[Normalised]	X	Y	Linear retardance δ [deg]	Optical rotation φ [deg]
	I_H	I_V	I_P	I_M					
H (0°)	199.7	0.5	100.1	106.6	$\mathbf{S_H} = \begin{bmatrix} 1 \\ 0.995 \\ -0.0325 \end{bmatrix}$	1.965	−0.1	13.09°	−1.47°
P (45°)	3900	3400	7200	72	$\mathbf{S_P} = \begin{bmatrix} 1 \\ 0.068 \\ 0.97 \end{bmatrix}$				
SLM at 25% Grey Scale									
H (0°)	198.6	3.2	127.6	78	$\mathbf{S_H} = \begin{bmatrix} 1 \\ 0.968 \\ 0.246 \end{bmatrix}$	1.899	0.314	22.24°	4.70°
P (45°)	3400	3900	7000	200	$\mathbf{S_P} = \begin{bmatrix} 1 \\ -0.0685 \\ 0.931 \end{bmatrix}$				
SLM at 50% Grey Scale									
H (0°)	139.4	61.4	176.2	20.5	$\mathbf{S_H} = \begin{bmatrix} 1 \\ 0.388 \\ 0.775 \end{bmatrix}$	1.177	1.070	53.73°	21.15°
P (45°)	2500	4600	6400	800	$\mathbf{S_P} = \begin{bmatrix} 1 \\ -0.295 \\ 0.789 \end{bmatrix}$				
SLM at 75% Grey Scale									
H (0°)	20	191.1	158	62	$\mathbf{S_H} = \begin{bmatrix} 1 \\ -0.811 \\ 0.435 \end{bmatrix}$	-0.520	1.170	72.71°	−33.27°
P (45°)	900	5900	4700	2700	$\mathbf{S_P} = \begin{bmatrix} 1 \\ -0.735 \\ 0.294 \end{bmatrix}$				
SLM at 100% Grey Scale									
H (0°)	6.7	200	143.5	78.4	$\mathbf{S_H} = \begin{bmatrix} 1 \\ -0.930 \\ 0.315 \end{bmatrix}$	-1.830	0.623	20.21°	−9.37°
P (45°)	2700	5100	175.6	7200	$\mathbf{S_P} = \begin{bmatrix} 1 \\ -0.308 \\ -0.901 \end{bmatrix}$				

The obtained variation of optical rotation (φ) and linear retardance (δ) with increasing grayscale percentages is plotted in Fig. 4.

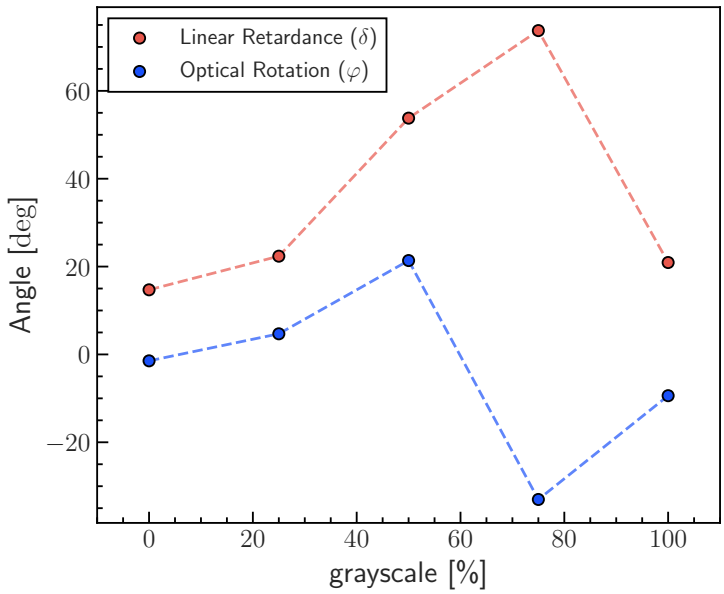


Figure 4: The figure shows the variation of **optical rotation** (φ) and **linear retardance** (δ) in Y-axis with increasing grayscale values of SLM in X-axis. [Since the corresponding errors are smaller than the minimum divisions in the graph, we have not plotted it]

5 Error Analysis

We calculate the maximum possible error for both δ and φ, considering the least count of photo-detector as $\Delta I = 0.1\mu$A.

5.1 Errors ΔX and ΔY

$$X = \left(\frac{I_H - I_V}{I_H + I_V}\right)_H + \left(\frac{I_P - I_M}{I_H + I_V}\right)_P$$

$$Y = \left(\frac{I_P - I_M}{I_H + I_V}\right)_H - \left(\frac{I_H - I_V}{I_H + I_V}\right)_P$$

It is easy to see that (maximizing errors by triangle inequality),

$$|\Delta X| \leq \Delta\left|\left(\frac{I_H - I_V}{I_H + I_V}\right)_H\right| + \Delta\left|\left(\frac{I_P - I_M}{I_H + I_V}\right)_P\right| \leq 2\Delta I\left[\frac{1}{(I_H + I_V)_H} + \left(\frac{I_P + I_M}{(I_H + I_V)^2}\right)_P\right],$$

$$|\Delta Y| \leq \Delta\left|\left(\frac{I_H - I_V}{I_H + I_V}\right)_P\right| + \Delta\left|\left(\frac{I_P - I_M}{I_H + I_V}\right)_H\right| \leq 2\Delta I\left[\frac{1}{(I_H + I_V)_P} + \left(\frac{I_P + I_M}{(I_H + I_V)^2}\right)_H\right],$$

5.2 Error in Linear Retardance (δ)

$$\delta = \cos^{-1}\left[\sqrt{X^2 + Y^2} - 1\right]$$

Taking the derivative after applying cosine to both sides:

$$-\sin(\delta)\,\mathrm{d}\delta = \cdot\frac{X\,\mathrm{d}X + Y\,\mathrm{d}Y}{\sqrt{X^2 + Y^2}}.$$

Since, $\sin(\delta) = \sqrt{1 - \cos^2(\delta)}$, and $\cos(\delta)$ can be expressed in terms of X and Y, we obtain:

$$\mathrm{d}\delta = \frac{-1}{\sqrt{1 - \left(\sqrt{X^2 + Y^2} - 1\right)^2}} \cdot \frac{X\,\mathrm{d}X + Y\,\mathrm{d}Y}{\sqrt{X^2 + Y^2}}.$$

Hence

$$\boxed{\Delta\delta \leq \frac{|\Delta X|}{\sqrt{1-\left(\sqrt{X^2+Y^2}-1\right)^2}} \cdot \left|\frac{X+Y}{\sqrt{X^2+Y^2}}\right|}$$

5.3 Error in Optical Rotation (φ)

From

$$\varphi = \frac{1}{2}\tan^{-1}\left(\frac{Y}{X}\right),$$

Differentiating,

$$\mathrm{d}\varphi = \frac{1}{2} \cdot \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \mathrm{d}\left(\frac{Y}{X}\right).$$

Now,

$$\mathrm{d}\left(\frac{Y}{X}\right) = \frac{X\,\mathrm{d}Y - Y\,\mathrm{d}X}{X^2}.$$

Therefore,

$$\mathrm{d}\varphi = \frac{1}{2} \cdot \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \frac{X\,\mathrm{d}Y - Y\,\mathrm{d}X}{X^2}.$$

$$\boxed{\Delta\varphi \leq \frac{1}{2} \cdot \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \left|\frac{X\,\Delta Y - Y\,\Delta X}{X^2}\right|}$$

5.4 Sources of Error

- The presence of stray light / multiple reflections from various optical surfaces could have affected the actual intensity measurements.
- Random electric noise and voltage fluctuations are inherent in the photodetector setup.
- Heating up of the SLM due to prolonged operation may have resulted in inconsistent grayscale values over the entire cross section of the beam.
- The theory assumes the use of *ideal* polarizers, however our polarizers possess a finite extinction ratio. This leads to residual transmission or *leakage* of orthogonal light components, meaning measured minima (such as I_V for a horizontally polarized input) remain non-zero. Thereby introducing a systematic bias into the measurement of the stokes parameters.
- Inadvertent human error, while adjusting for the desired orientation of polarizers is another major source of uncertainty.

6 Discussion

While the Jones matrix formalism provides the total birefringence accumulated in the phase of the light, it fails to distinguish between the *dynamical* phase and the *geometric* (Pancharatnam-Berry) phase. Separating these two components solely through interferometry presents significant technical challenges.

In contrast, the Stokes-Mueller formalism facilitates the isolation of these phases. By measuring light intensities across multiple polarization axes to determine the output Stokes vector, the complete Mueller matrix of the system can be characterized. This approach enables the precise extraction of the effective geometric birefringence. Experiment wise, it would have been interesting to repeat this experiment with a green laser (or any other wavelength) and compare the results.

7 Conclusion

Through our experiment, we were able to successfully investigate the properties of SLM, specifically how it changes with varying grayscale values. To achieve this, we computed the linear retardance and optical rotation of the incident light beam at 0, 25, 50, 75 and 100 % mode of grayscale. The measured values are tabulated below.

Grayscale	**Linear Retardance** $\delta \pm \Delta\delta$ (deg)	**Optical Rotation** $\varphi \pm \Delta\varphi$ (deg)
0%	13.09 ± 0.25	-1.47 ± 0.01
25%	22.24 ± 0.18	4.70 ± 0.02
50%	53.73 ± 0.10	21.15 ± 0.03
75%	72.71 ± 0.03	-33.27 ± 0.01
100%	20.21 ± 0.10	-9.37 ± 0.01

Table 2: Measured values of Linear Retardance and Optical Rotation at different grey scale levels.

8 Script

```
#
import math
import numpy as np

print("Error calculation for Linear Retardance and Optical Rotation\n")

# detector least count
delta_I = float(0.1)


# gscale = [0,25,50,75,100]

# measured intensities
print('For H (muA)')
IH1 = float(input("I_H: "))
IV1 = float(input("I_V: "))
IP1 = float(input("I_P: "))
IM1 = float(input("I_M: "))

print('For P (muA)')
IH2 = float(input("I_H: "))
IV2 = float(input("I_V: "))
IP2 = float(input("I_P: "))
IM2 = float(input("I_M: "))

#X1, Y1 calculation

X = (IH1 - IV1)/(IH1 + IV1) + (IP2 - IM2)/(IH2 + IV2)
Y = (IP1 - IM1)/(IH1 + IV1) - (IH2 - IV2)/(IH2 + IV2)

print('Calculated X = ', X)
print('Calculated Y = ', Y)
# # measured X1 and Y1
# X = float(input("X1: "))
# Y = float(input("Y1: "))

lin_retard = np.arccos(np.sqrt(X**2 + Y**2)-1)
opt_rot    = 0.5* np.arctan(Y/X)

print(f'Linear Retardance [deg] = {np.degrees(lin_retard ):.2f}')
print(f'Optical Rotation [deg] = {np.degrees(opt_rot):.2f}')

# ----  ----
deltaX = 2 * delta_I * ( abs(IH1 + IV1)/((IH1 + IV1)**2)  + abs(IP2 + IM2)/((IH2 + IV2)
    **2) )
deltaY = 2 * delta_I * ( abs(IH2 + IV2)/((IH2 + IV2)**2)  + abs(IP1 + IM1)/((IH1 + IV1)
    **2) )
# ---- Linear retardance error ----
r = math.sqrt(X**2 + Y**2)

delta_delta = (deltaX/ np.sqrt(1 - (r - 1)**2)) * abs((X + Y) / r)

# ---- Optical rotation error ----
delta_psi =  (0.5 / (1 + (Y/X)**2)) * abs((X*deltaY + Y*deltaX) / (X**2))

# ---- Output ----
print("\nResults")
print("----------------------")

print(f"Error in Linear Retardance Dlin = {np.degrees(delta_delta):.2f} deg")
print(f"Error in Optical Rotation Dopt = {np.degrees(delta_psi):.2f} deg")
```

References

- Lab manual
- Polarisation Theory [https://www.youtube.com/watch?v=RowMxWt4mVE]
- SLM Theory [https://www.youtube.com/watch?v=-KzEcJg1wWM; https://www.intechopen.com/chapters/83627]
- On stokes vector measurements [https://pol3he.sites.wm.edu/wp-content/uploads/sites/50/2020/01/measuring-Stokes-parameters.pdf]