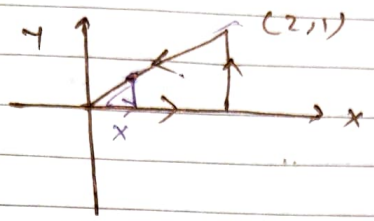
Tutorial-3

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$\vec{v} = (2x+y^2)\hat{i} + (3y-4x)\hat{j}$

1)

$\oint_c \vec{v} \cdot d\vec{r}$



$d\vec{S} = |dS|\,\hat{n}$

$= dx\,dy\,\hat{k}$

$= dx\,dy\,\hat{k}$.

$x = 2y$

$y = x/2$.

By Stokes Theorem we know.

$$\int_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{S} = \oint_c \vec{v} \cdot d\vec{r}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+y^2 & 3y-4x & 0 \end{vmatrix}$$

Our only concern is $(\vec{\nabla} \times \vec{v})_z$.

$= \hat{k}\,(-4) - (2y)$

$= 0 - (4+2y)\,\hat{k}$

$\therefore \int_S -(4+2y)\,dx\,dy$.

$= -\int_0^2 \left[\int_0^{x/2} (4+2y)\,dy\right] dx$.

$= -\int_0^2 \left[(4y + y^2)_0^{x/2}\right] dx$

$= -\int_0^2 \left(2x + \frac{x^2}{4}\right) dx$

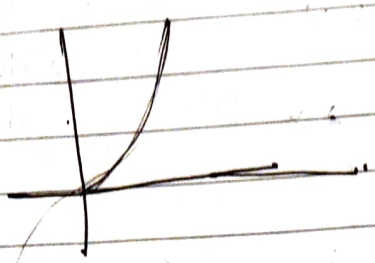
$= -\left[x^2 + \frac{1}{12}x^3\right]_0^2$.

$= -\left[\frac{16}{3} + \frac{1}{12} \times 8\right] = -\frac{14}{3}$.

Ans $= \frac{-14}{3}$

2) $\vec{v} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$,

Find $\int_c \vec{v} \cdot d\vec{r}$ along C, $y = x^3$



$d\vec{r} = dx\hat{i} + dy\hat{j}$

$$\therefore \int_c \vec{v} \cdot d\vec{r} = \int (5xy - 6x^2)\,dx + \int (2y - 4x)\,dy$$

$y = x^3$

$$= \int_1^2 (5x^4 - 6x^2)\,dx + \int_1^8 (2y - 4y^{1/3})\,dy$$

$dy = 3x^2\,dx$

$$= \int_1^2 (5x^4 - 6x^2)\cdot dx + \int (2x^3 - 4x)3x^2 \cdot dx$$

$$= \int_1^2 (6x^5 + 5x^4 - 12x^3 - 6x^2)\,dx$$

$$= [x^6 + x^5 - 3x^4 - 2x^3]_1^2$$

$$= [(64-1) + (32-1) + (-48+3) + (-16+2)]$$

$$= 94 - 59$$

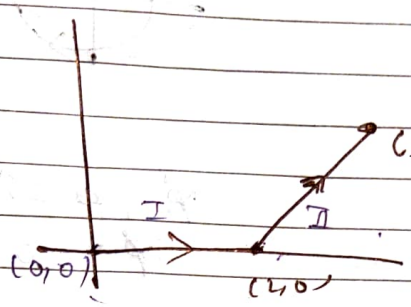
$$= 35$$

Ans: 35

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3) $\vec{v} = (2x+y)\hat{i} + (3y-x)\hat{j}$

$\int_C \vec{v} \cdot d\vec{r}$



$$\int_{C_I} \vec{v} \cdot d\vec{r} = \int_0^2 (2x+0) \cdot dx + \int_0^0 (3y-x)\, dy$$

$$= 4$$

$$\int_{C_{II}} \vec{v} \cdot d\vec{r} = \int_2^3 (2x+y)\, dx + \int_0^2 (3y-x) \cdot dy.$$

Eq of line

$$\frac{y_2-y_1}{x_2-x_1} = \frac{y_2-y}{x_2-x}$$

$$\Rightarrow \frac{2-0}{3-2} = \frac{2-y}{3-x}$$

$$6-2x = 2-y$$

$$y = 2x-4$$

Am II

$$\therefore = \int_2^3 (2x+0-y) \cdot dx + \int_0^2 \left(3y - \left(\frac{y+4}{2}\right)\right) dy$$

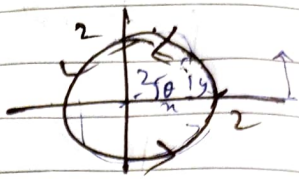
$$= \int_2^3 (2x-4x) \cdot dx + \int_0^2 \left(\frac{5y}{2}+2\right) \cdot dy$$

$$= 2x^2 - 4x \Big|_2^3 + \left[\frac{5}{4}y^2 + 4y\right]_0^2 = \frac{5}{4} + \text{Ans}$$

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4) $\vec{F} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$

$\oint \vec{F} \cdot d\vec{r}$



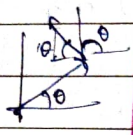
~~$x^2 + y^2 = 4$~~

~~$d\vec{r} =$~~ $\vec{r} = 2\hat{r} + \theta\hat{\theta}$

$d\vec{r} = 0 + d\theta\,\hat{\theta}$

$y = 2\sin\theta$

$x = 2\cos\theta$



$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ ~~$-\sin\theta$~~

$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ ~~$+\cos\theta$~~

$\cos\theta\,\hat{\theta} + (\sin\theta)\,\hat{r} = \hat{j}$ ✓

$(-\sin\theta)\,\hat{\theta} + (\cos\theta)\,\hat{r} = \hat{i}$ ✓

$\vec{F} = (2\sin\theta - 4\cos\theta)\left((-\sin\theta)\,\hat{\theta} + (\cos\theta)\,\hat{r}\right)$

$+ (6\cos\theta + 4\sin\theta)\left((\cos\theta)\,\hat{\theta} + (\sin\theta)\,\hat{r}\right)$

$\therefore \oint_0^{2\pi} \vec{F} \cdot d\vec{r} = -2\sin^2\theta + 4sc + 6c^2 + 4sc$

$= \int_0^{2\pi} (-2\sin^2\theta + 8\sin\theta\cos\theta + 6\cos^2\theta)\, d\theta$

$Am = 4\pi$

$I = 4\left[\int_0^{\pi/2} (-2\sin^2\theta + 6\cos^2\theta)\, d\theta\right]$

$I = 4\left[\int_0^{\pi/2} (-2\cos^2\theta + 6\sin^2\theta)\, d\theta\right]$

$2I = 16\left[\int_0^{\pi/2} \text{~~$-2+6$~~}\, d\theta\right]$

$I = 4\pi$

$2I = 16 \times \frac{\pi}{2}$