Summary Notes on

ES 2102

Hydrology and Geodynamics



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by

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Contents

1 Week-1.1 **1**
- 1.1 What are the requirements of a soil to hold water? . . . 1
- 1.2 Meandering Systems . . . 1
- 1.3 Preliminary definitions . . . 1
- 1.4 Aquifer . . . 2
- 1.5 How does water flow in an aquifer? . . . 3

2 Week-1.2 **4**
- 2.1 Darcy's Law . . . 4
- 2.2 Reynolds Number (R) . . . 4
- 2.3 Storativity or Coefficient of Storage . . . 5
- 2.4 Specific Yield . . . 5
- 2.5 Specific Retention . . . 5
- 2.6 Transmissivity . . . 5
- 2.7 Resistance . . . 6
- 2.8 Groundwater Flow Equation (Confined Aquifer) . . . 6

3 Week-1.3 **8**
- 3.1 Unidirectional Steady State flow for Confined Aquifer . . . 8
- 3.2 Unidirectional Steady State flow for Unconfined Aquifer . . . 8
- 3.3 Radial Flow in Confined Aquifer . . . 8
- 3.4 Radial Flow in Unconfined Aquifer . . . 9
- 3.5 Theis theory for Transient Flow . . . 10
- 3.6 Graphical Approach - Cooper Jacob Method . . . 11
- 3.7 Distance Drawdown Method . . . 12
- 3.8 Theis Type Curve Method . . . 13

4 Week-1.4 **16**
- 4.1 Determining residence time of water in aquifer- ${}^{14}_{6}C$ Method . . . 16
- 4.2 Factors affecting ${}^{14}C$ composition . . . 18
- 4.3 $CaCO_3 - CO_2 - H_2O$ Equilibrium . . . 18

5 Week-1.5 **20**
- 5.1 ${}^{3}_{1}H$ method to determine infiltration rate of Aquifer . . . 20
- 5.2 Arsenic Problem . . . 21
  - 5.2.1 How is As mobilised in water? . . . 21
  - 5.2.2 Why are Deep Aquifers less contaminated compared to shallow aquifers? . . . 21

6 Week-1.6 **22**
- 6.1 Sand Tank Model . . . 22
- 6.2 Analog Model . . . 22
- 6.3 Resistivity Method . . . 23
- 6.4 Finite difference Method . . . 26

7 Exercises **27**

8 Week-2.1 **28**
8.1 Determination of root of any elevated topography . . . 28
8.1.1 Calculation of root size . . . 29
8.2 Erosion of Mountains . . . 29
8.3 Pratt-Hayford Isostasy Model . . . 30
8.4 Airy–Heiskanen Isostacy Model . . . 31

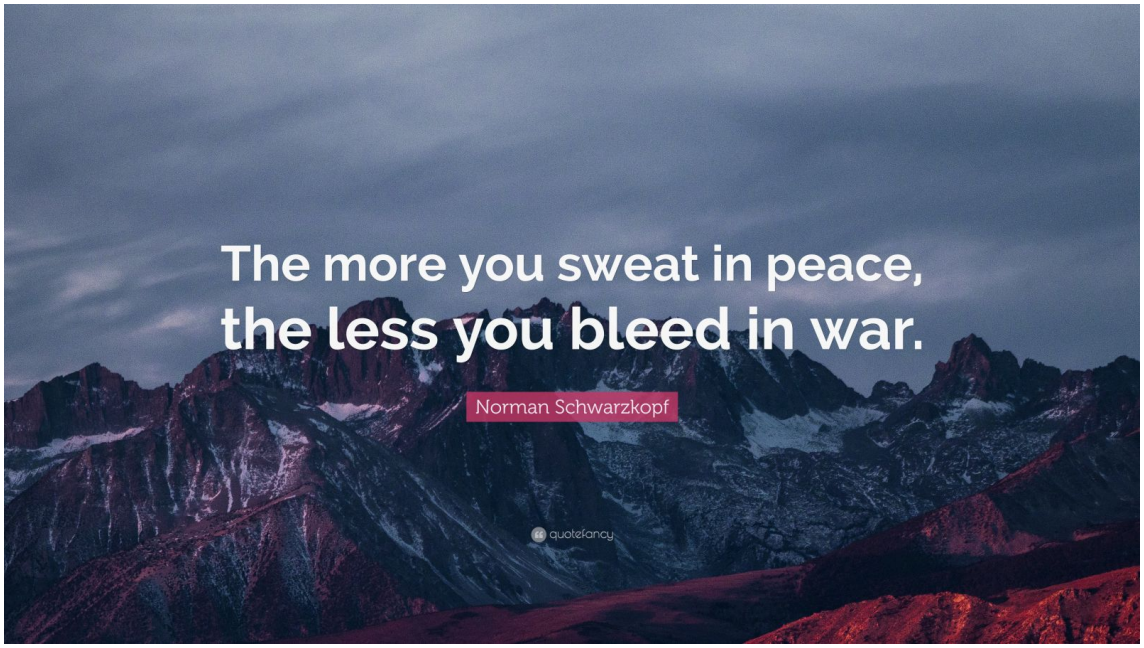
9 Week 2.2 **32**
9.1 Lithospheric Mantle Slab . . . 32
9.2 Topography of Ocean Basin . . . 33

10 Week 2.3 **34**
10.1 Estimation of Extensional Setting . . . 34
10.2 Estimation of Contractional Setting . . . 35

11 Week 2.4 **36**
11.1 Gravity Anomaly and Corrections . . . 36
11.1.1 Free Air Anomaly . . . 36
11.1.2 Bouguer Anomaly . . . 36
11.1.3 Anomalous Mass . . . 37
11.2 Gravity Modeling . . . 38

12 Week 2.5 **39**
12.1 Plate is continuous . . . 40
12.2 Plate is broken . . . 40

13 Week 2.6 **42**
13.1 Deviatoric Stress . . . 42
13.2 Passive Margin . . . 42


The more you sweat in peace,
the less you bleed in war.
Norman Schwarzkopf
quotefancy

Module 1: Groundwater Hydrology

1 Week-1.1

Hydrology is the scientific study of the distribution, movement, and properties of water in the Earth's atmosphere and on its surface. It plays a crucial role in understanding water resources, managing water-related issues, and predicting the behavior of water systems. In this course we shall learn in details about aquifers.

1.1 What are the requirements of a soil to hold water?

- **Soil type(size) must be clay.**
- Ponds exist in regions where water holding capacity is high.
- Clay is composed of *Alumina Hydrous Silicate*, which has a layered structure, so it's slippery
- Clay has high water holding capacity, because it's pore spaces are not connected. The angularity[2] is high, so pieces of clay particles are stuck together (S.A/Vol ↑), and doesn't allow water to percolate.
- **Sand** particles are quite large, so their angularity is lost by collision during transportation. So their packing is very loose and hence water seeps through it.

1.2 Meandering Systems

If we drill a vertical hole through the soil, we shall find alternate layers of sand and silt respectively. **Why?**

In river, their are mainly 2 types of deposits.

- Channel deposits- Consists of **sand**, as water current has high energy midway in the river, so it is able to transport and sediment heavier particles(sand)
- Fluvial deposits- Consists of **clay/silt**. Found where slow water current is present or floodplains. Highly fertile.

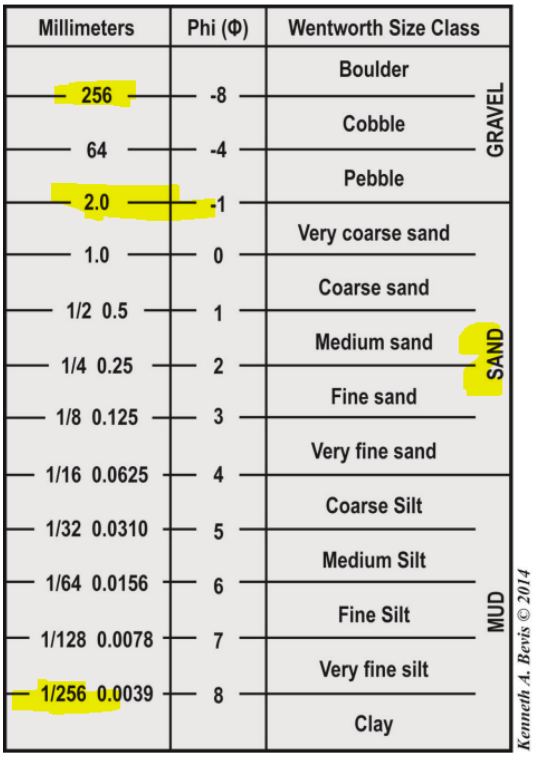
1.3 Preliminary definitions

1. Porosity= $\left(\frac{\text{Vol of voids}}{\text{total vol of grain}} = \text{Void ratio}\right) \times 100\%$. Clay > Sand.
2. Permeability- Ease with which water can flow ≡ Interconnectivity of pore spaces.
3. Textural classification of grain.

$$\Phi = -\log_2 d \ (mm)$$

where d= diameter of grain.

[2]Number of edges in a grain



1.4 Aquifer

An aquifer is an underground layer of permeable rock, soil, or sediment (geological material) that *stores and transmits water*. **Sand** is an ideal material for aquifer, because pore spaces are interconnected. Hence, digging a borewell and pumping out water will not result in vacuum.

More sand $\implies$ good aquifer.

On the order of *sand content*, Aquifer is of the following type-

- Aquifer
- Aquitand
- Aquiclude
- Aquifuge

decreasing

Unsaturation Zone consists of air+mud+fluid+sand+...

P_H= Hydrostatic pressure.

In an unconfined aquifer, groundwater is in direct contact with the atmosphere through the open pore spaces of the overlying soil or rock.

The **piezometric surface**, also known as the groundwater table or phreatic surface, is an imaginary surface within an aquifer that represents the level to which water will rise under hydrostatic pressure due to groundwater. In simpler terms, it is the level to which water will naturally rise in wells and boreholes that penetrate the aquifer.

Note that: if confined aquifer is oversaturated then piezometric surface may not be confined within the aquifer.

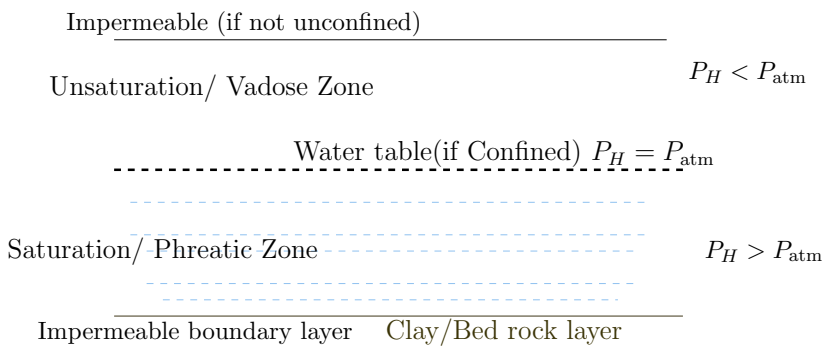


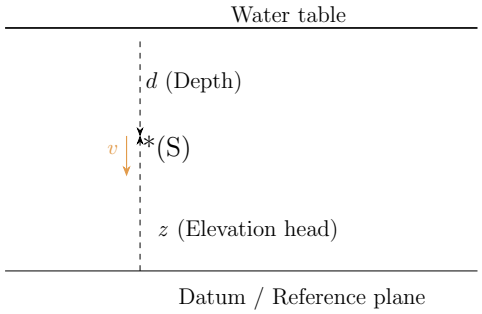
Figure 1: Internal Structure of Aquifer

1.5 How does water flow in an aquifer?

`(read pg22 of Hydrology slide)`
It is determined by a concept called **Hydraulic head**, which is related to energy. Expressed by h, dimensions of length.

Hydraulic head = elevation head + pressure head + velocity head.

$$h = z + \frac{P}{\rho_w g} + \frac{v^2}{2g}$$



Now consider a point $*(S)$ inside the saturation zone of an aquifer, at a depth d. The reference plane can be taken as a sea level. At $*$, let velocity of water be v, then $v^2 = 0 + 2gd$, then **velocity head** is defined as

$$d_v \equiv d = \frac{v^2}{2g}$$

Again, excess pressure at this point is $P = \rho_w g d$, then **pressure head** is defined as

$$d_P \equiv d = \frac{P}{\rho_w g}$$

Now, if $*$, is at water level, then $d_P, d_v = 0$ ($\because v \simeq 0$).
So, **Hydraulic head = Elevation head =** z.

2 Week-1.2

2.1 Darcy's Law

It states that in an incompressible fluid under laminar flow, Discharge (Q) is proportional to the Area of cross section and hydraulic head difference.

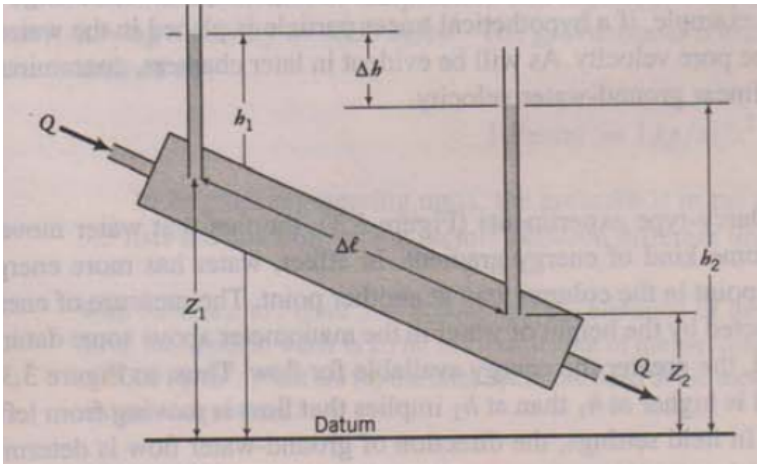


Figure 2: Darcy's Law set up

$$Q = \frac{dV}{dt} = KA\ i = KA.\frac{h_1 - h_2}{\Delta L} = -KA\frac{dh}{dl}$$

- $i = -\frac{dh}{dl}$ is hydraulic gradient.
- K = Hydraulic Conductivity/ Permeability.(Specific for a particular fluid).
- A = Area of cross section.
- $q = Q/A$ is called *Darcy velocity*. Flow through unit cross-section Area.

Intrinsic permeability(k) is a property which is fluid independent,

$$k = \frac{K\mu}{\rho g}$$

μ is coefficient of viscosity.

Note that: only in Laminar flow, discharge is $\perp$ to A,(Expt Darcy's Law has been found to be valid till $R \leq 10$). However in *Turbulent flow*, vertical components arise due to formation of *eddies*, and so discharge is not proportional to the A.

2.2 Reynolds Number (R)

$$R = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

If $R < 1 \implies$ laminar flow, otherwise turbulent. Numerically,

$$R = \frac{\rho v d}{\mu}$$

2.3 Storativity or Coefficient of Storage

It is defined as the volume of water that an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. Large S, means capacity to hold large volume of water, which means it is healthy aquifer. Otherwise poor health.

$$S = \frac{V}{A \times (Head)}$$

Specific Storage is defined as,

$$S_s = \frac{S}{\text{thickness of aquifer}(b)}$$

Note that head, is related to an imaginary surface, whereas thickness is something physical.

Problem 1 A water saturated medium has a hydraulic conductivity of 20 m/day. What will be the intrinsic permeability of that medium? The density and dynamic viscosity of water are 998.2 kg/m^3 and 1.002×10^{-3} kg/m.sec.

Solution: Given, $K = 20$ m/day, $\mu = 1.002 \times 10^{-3}$ kg/m.sec, $\rho = 998.2$ kg/m^3, g= 9.8 m/s^2.

Then $k = \frac{K\mu}{\rho g}$

2.4 Specific Yield

$$S_y = \frac{V_d}{V} = \frac{\text{water drained out by gravity}}{\text{Total volume of material}}$$

2.5 Specific Retention

$$S_r = \frac{V_r}{V} = \frac{\text{vol. of water retained after gravitational drain-out}}{\text{Total volume of material}}$$

Storage in an *unconfined aquifer* is defined by it's storativity,

$$S' = S_y + bS_s$$

Note that:

$$S_y + S_r = \eta \equiv \text{Porosity } \%$$

This is because all of the water in an aquifer is either held in the larger pore spaces or in the smaller pore spaces. So, their sum directly gives a measure of their porosity.

2.6 Transmissivity

The transmissivity is a measure of how much water can be transmitted horizontally, such as to a pumping well. For an aquifer of thickness b, it is given as-

$$T_i = K_{h_i} b_i$$

K_h is horizontal hydraulic conductivity

2.7 Resistance

The resistance of a vertical flow, of i^{th} soil layer, of saturated thickness b_i, and K_{v_i} vertical hydraulic conductivity is

$$R_i = \frac{b_i}{K_{v_i}}$$

Note that: Normally in an *virgin* aquifer these values are constant. But, if we keep on extracting groundwater from it, then these values will change. Slow change $\implies$ good health, drastic change $\implies$ poor health.

Problem 2 After a soil sample is drained by gravity the weight of the soil sample is 85gm. After the sample is over dried, the sample weighs 80gm. The bulk density of the wet soil is 1.65 gm/cc and ρ_w = 1gm/cc. The amount of water drained by gravity is 20gm. Calculate the porosity of soil.

Solution: Total volume of material = $V = (85 + 20)/1.65 = 63.63$ cc
Volume of water drained, $V_d = 20/1 = 20$cc
Volume of water retained after draining = $V_r = (85 - 80)/1 = 5$cc
So porosity,

$$\eta = S_y + S_r = \frac{V_d}{V} + \frac{V_r}{V} = \frac{20 + 5}{63.63} = 0.3928$$

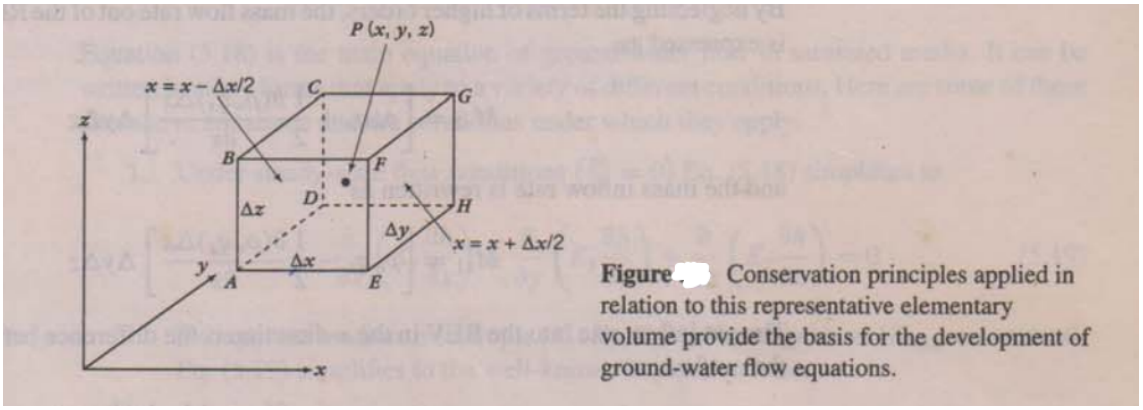


Figure Conservation principles applied in relation to this representative elementary volume provide the basis for the development of ground-water flow equations.

2.8 Groundwater Flow Equation (Confined Aquifer)

Here we assume a *confined* aquifer, which is infinitely extending. Then by principle of mass conservation we have-
Mass influx - Mass outflux(rate) = Change of mass of water storage with time.

$$(\dot{M}_{x1} - \dot{M}_{x2}) + (\dot{M}_{y1} - \dot{M}_{y2}) + (\dot{M}_{z1} - \dot{M}_{z2}) = \frac{\partial}{\partial t}(\Delta x \Delta y \Delta z \ \eta \rho_w)$$

where, $\dot{M}_i = \rho_w q_i \Delta A_i$,
ΔA_i= cross section perpendicular to flow in direction i. And, $q_x = -K_x \frac{\partial h}{\partial x}$ and so on...

Solving and assuming *Isotropic* distribution, so, $K_x = K_y = K_z = K$ and ρ_w being spatially constant, we arrive at

$$\nabla^2 h = -\frac{S_s}{K}\frac{\partial h}{\partial t} = -\frac{S}{T}\frac{\partial h}{\partial t}$$

At **steady state** scenario,

$$\nabla^2 h = 0$$

If a groundwater source exists, along with an anisotropic distribution then then, the above equation is modified as,

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial y}\left(K_z\frac{\partial h}{\partial z}\right) + Q(x,y,z,t) = -S_s\frac{\partial h}{\partial t}$$

Note that:

$$\frac{1}{\rho_w}\frac{\partial \rho_w \eta}{\partial t} = S_s\frac{\partial h}{\partial t}$$

Problem 3 Why the groundwater flow equation of a saturated aquifer differs from that of unsaturated aquifer?

Solution: In the case of groundwater flowing in saturated aquifer, all the pores will be occupied by water and it will be a homogeneous mixture of water and sand. However, in unsaturated aquifer, the pore spaces contain both water and air. The degree of saturation (the ratio of the volume of water to the total pore space) is less than 1. It's groundwater flow equation (Richards Equation))includes terms for capillary pressure and water retention, making it more complex than the groundwater flow equation for saturated aquifers.

3 Week-1.3

3.1 Unidirectional Steady State flow for Confined Aquifer

We have, $\frac{\partial^2 h}{\partial x^2} = 0 \implies h = c_1 x + c_2$.
Subjecting to constraints, $h(0) = h_0,\ h(L) = h_L$

$$h(x) = \left(\frac{h_L - h_0}{L}\right)x + h_0$$

The darcy velocity then becomes, $q = -K\frac{\mathrm{d}h}{\mathrm{d}x} = -K(\frac{h_L - h_0}{L}) = \text{constant}$

3.2 Unidirectional Steady State flow for Unconfined Aquifer

In this case, the groundwater flow equation is

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial z}\right) = 0$$

For unidirectional flow, $\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = 0 \implies h^2(x) = c_1 x + c_2$.
Subjecting to constraints, $h(0) = h_0,\ h(L) = h_L$

$$h(x) = \sqrt{\left(\frac{h_L^2 - h_0^2}{L}\right)x + h_0^2}$$

And the darcy velocity, $q(x) = \frac{-K}{2L}\frac{h_L^2 - h_0^2}{h(x)}$

3.3 Radial Flow in Confined Aquifer

Radial flow is the flow of water, when pumped out by hand pump.
Let the confined aquifer has thickness= b

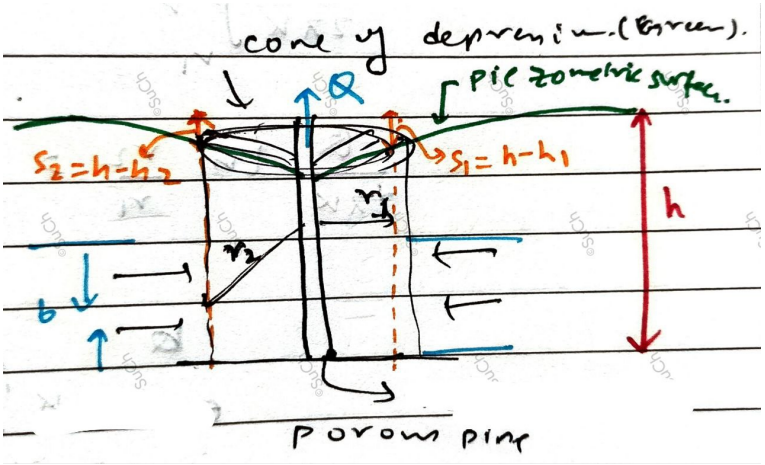


Figure 3: Confined Aquifer structure

Then acc. to Darcy's Law

$$\begin{aligned} Q &= KA\frac{\partial h}{\partial r} \\ &= K(2\pi rb)\frac{\partial h}{\partial r} \\ \Rightarrow \int_{h_1}^{h_2} \mathrm{d}h &= \frac{Q}{2K\pi b}\int_{r_1}^{r_2}\frac{\mathrm{d}r}{r} \end{aligned}$$

Here we define the drawdown length of the piezometric surface (see fig), $s_i \equiv h - h_i$, h_i is the observed value of hydraulic head at a distance r_i from the borewell. Substituting the values we obtain,

$$Q = 2\pi bK\frac{s_1 - s_2}{\ln r_2/r_2}$$

3.4 Radial Flow in Unconfined Aquifer

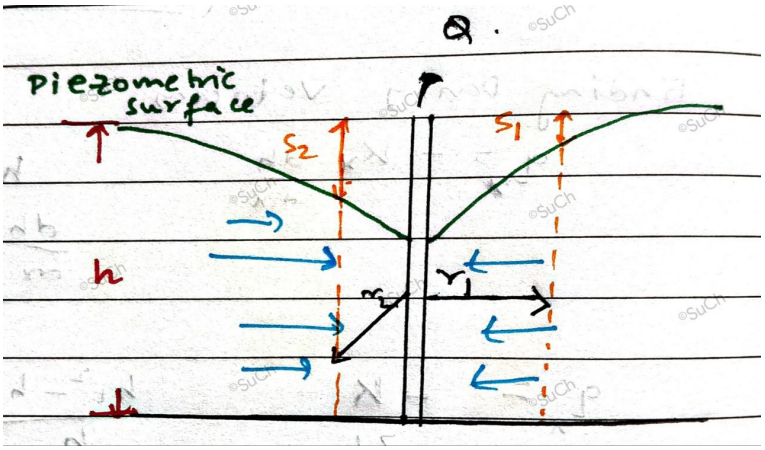


Figure 4: Unconfined Aquifer structure

Similarly as before, only here the area of cross section, $A = 2\pi rh$

$$\begin{aligned} Q &= KA\frac{\partial h}{\partial r} \\ &= K(2\pi rh)\frac{\partial h}{\partial r} \\ \Rightarrow \int_{h_1}^{h_2} h\mathrm{d}h &= \frac{Q}{2K\pi b}\int_{r_1}^{r_2}\frac{\mathrm{d}r}{r} \end{aligned}$$

Solving,

$$Q = \pi K\frac{h_1^2 - h_2^2}{\ln r_2/r_2}$$

Problem 4 Two rivers 1000m apart penetrate a confined aquifer 20m thick. The hydraulic conductivity is 20m/day. The stages of the two rivers are 500m and 495m above sea level, respectively. What is the Darcy flow velocity for groundwater in the aquifer? Now, assume that the aquifer is unconfined, calculate the hydraulic head in the aquifer at midway between the river.

Solution: (i) $q = -K\frac{\Delta h}{\Delta l} = -20 \times \frac{495-500}{1000} = 0.1$m/day. **Ans**

(ii) To find $q(L/2) = \frac{-K}{2L}\frac{495^2-500^2}{h(L/2)}$
We have,

$$h(x)^2 = \left(\frac{h_L^2 - h_0^2}{L}\right)x + h_0^2$$

$$\therefore h_M \equiv h(L/2) = \sqrt{\left(\frac{495^2 - 500^2}{L}\right)L/2 + 500^2} \simeq 497.5\ m$$

Putting value, $q_M = 0.1$m/day **Ans**

Problem 5 A fully penetrating well of 30 cm diameter is installed in *confined* aquifer of 15 m thickness, is discharged at the rate of 56 m^3/hour. Calculate the hydraulic conductivity of aquifer, if draw-down in the piezometers located at the radial distance of 10 and 20 m from the well is 2.4 and 0.75 m, respectively under steady-state condition.

Solution: Given $(r_1, s_1) = (10, 2.4)$ and $(r_2, s_2) = (20, 0.75)$, b = 15m, Q= 56 m^3/hour. So,

$$Q = 2\pi bK\frac{s_1 - s_2}{\ln r_2/r_2} \implies 56 = 2\pi \times 15 \times K \times \frac{2.4 - 0.75}{\ln 20/10}$$

Solving, $K = 0.2495$ m/hr. **Ans**

3.5 Theis theory for Transient Flow

Theis theory lets us evaluate the behaviour of a well pumping in a *confined* aquifer that is under non-steady state conditions. Here we consider cylindrical coordinate system (r, ϕ, z), where we consider the flow as azimuthally symmetric, and assuming $h = f(r)$ only.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 h}{\partial \phi^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$$\therefore \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}h + \frac{\partial^2 h}{\partial r^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

The Boundary conditions are:

1. $h(r, 0) = h_o$
2. $h(\infty, t) = h_o$
3. $\lim_{r\to\infty}(r\frac{\partial h}{\partial r}) = \frac{Q}{2\pi T}$ (`Derive it!`)

The solution of this equation is,

$$h_0 - h \equiv s = \frac{Q}{4\pi T}w(u)$$

where Q is the pumping rate and T is transmissivity of the aquifer. The well function $w(u)$ is given by-

$$w(u) = \int_u^\infty \frac{e^{-y}}{y} \mathrm{d}y \ ; \ u = \frac{r^2 S}{4Tt}$$

Note that:

1. r = radial distance of observational well from a pumped well
2. t= time since pumping started
3. h = usual hydraulic head..

`Learn how to read` $w(u)$ `table and plot the` ***type*** `curve.`
This solution is based on the following assumptions:

1. The pumping well is fully penetrating, constant discharge rate, and negligible diameter and storage.
2. It is an infinitely extending, homogeneous, isotropic and confined aquifer.
3. All water pumped by the well comes from the storage and is discharged instantaneously with the decline in head.

3.6 Graphical Approach - Cooper Jacob Method

This method is used to calculate the two fundamental constants for an aquifer, namely Storativity(S) and Transmissivity(T).
We have, the well function, $w(u) \simeq -0.577216 - \ln u = \ln \frac{0.561}{u}$.
From drawdown formula $\equiv s = \frac{Q}{4\pi T} w(u) \implies s = \frac{Q}{4\pi T} \ln \frac{0.561}{u}$
Further, $1/u = \frac{4tT}{r^2 S}$, combining the equations we get

$$s = \frac{Q}{4\pi T}\left(\ln t + \ln \frac{2.25T}{r^2 S}\right)$$

Plotting it on (*arithmetic scale*) Y axis(s) and (*logarithmic scale*) X-axis (t- time through which pumping was conducted), we expect a line with +ve slope, but with an X-intercept corresponding to time- t_o.

Step 1 Select 2 points on the graph, atleast 1 log cycle apart. Say (s_1, t_1) & (s_2, t_2). Compute, Δs and $\frac{t_1}{t_2}$. Preferably $t_2 = 10t_1$.

Step 2 Find t_o. This represents the time taken to reach drawdown in the observational well. The values of Q, r should be known beforehand.

Note that: `Derive the formulas that we used!`

$$T = \frac{Q}{4\pi \Delta s} \ln t_1/t_2 \quad \& \quad S = 2.25 \frac{T t_o}{r^2}$$

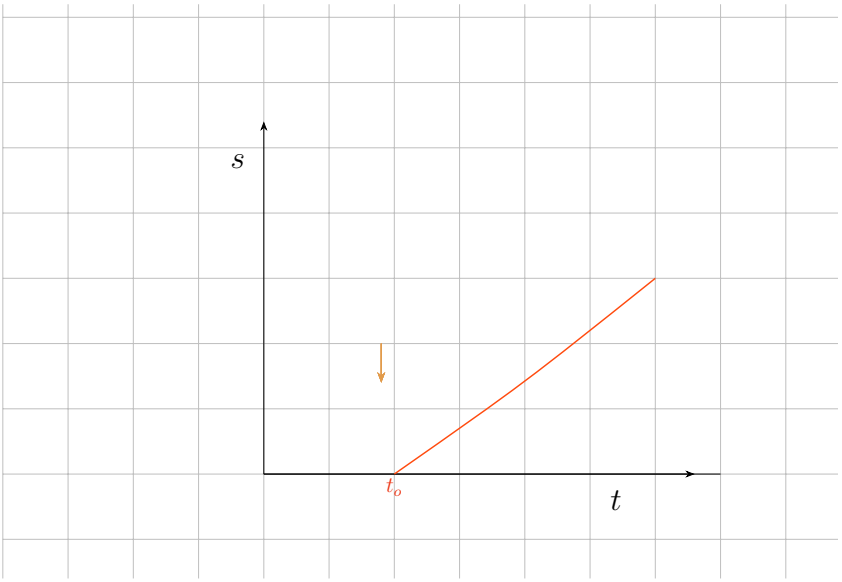


Figure 5: Straight line plot

3.7 Distance Drawdown Method

It is obtained by finding the values of drawdown length at several distances after an equal drawdown time.

$$s = \frac{Q}{4\pi T}\left(\ln \frac{2.25Tt}{r^2 S}\right)$$

After taking the difference between 2 readings from graph,

$$s_1 - s_2 = \frac{2.3Q}{2\pi T} \log \frac{r_2}{r_1}$$

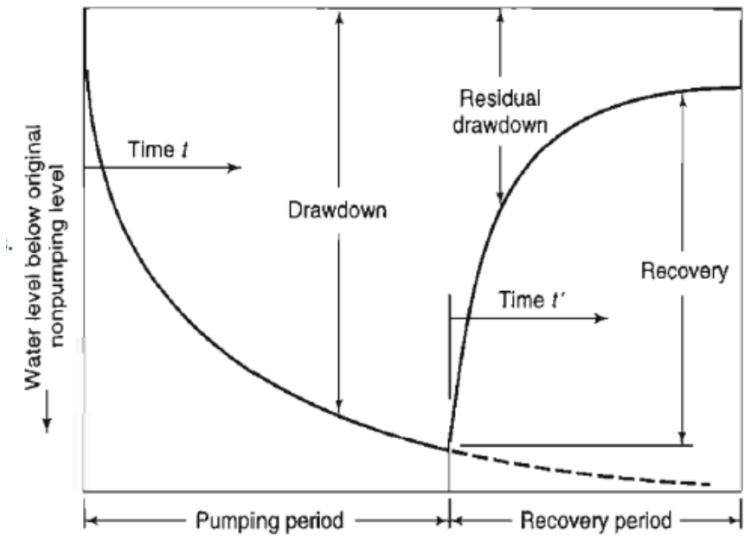


Figure 6: Plot showing the variation of drawdown length during pumping and non-pumping (recovery) period.

Observe Fig 6

- When groundwater is pumped from an aquifer the water table or piezometric surface in the aquifer drops temporarily ($h_i \downarrow \therefore s_i = h - h_i \uparrow$). This temporary drop is known as *transient drawdown*.
- But, if pumping continues over an extended period, the aquifer may not fully recover to its original water level during non-pumping periods. This is due to the fact that the rate of recharge (natural replenishment of the aquifer) may not be sufficient to compensate for the rate of extraction.
- When pumping stops, the time from t' is known as *recovery period* of aquifer.
- The *residual drawdown* measurement at any time during the recovery period is the difference between the observed water level and the prepumping static water level. It is a measure of the aquifer's ability to recover from pumping. Thus greater residual drawdow means the aquifer is stressed.

3.8 Theis Type Curve Method

In this method we shall learn how to determine the Storativity and Transmissivity of a given confined aquifer. The steps are:

Step 1 Plot the graph on Fig 8. This is is the universal type curve. Take the values of only the fist 3 rows from the well function table.

Step 2 Plot the graph on Fig 9. This is unique for a given aquifer. Remember to join the points with the *best fitting smooth curve*.

Step 3 Take a tracing/butter/ghee paper and put in exactly on top of the graph area of Fig 9. And trace the curve.

Step 4 Then take the tracing paper and put it exactly as shown in Fig 7. Note that, the axes of the graph MUST be aligned parallel to the sides of the tracing paper, do *not* rotate the paper. Make only horizontal and vertical movements parallel to the axes of Fig 8.

Step 5 Once you get a point, draw straight lines which will cut the axes of your graph in Fig 8. Take the same tracing paper, and put it back as it was on graph in Fig 9. Mark the coordinates corresponding to the points of intersection.

Step 6 Hopefully you will find the points $(w(u), \frac{1}{u}, s, t)$ corresponding to each intersection points. The values of Q and r, shall be given beforehand.4

Step 7 Use $T = \frac{Q}{4\pi s} w(u)$ and $S = \frac{4Ttu}{r^2}$, to get the values and compare them. Remember to take care of the units!

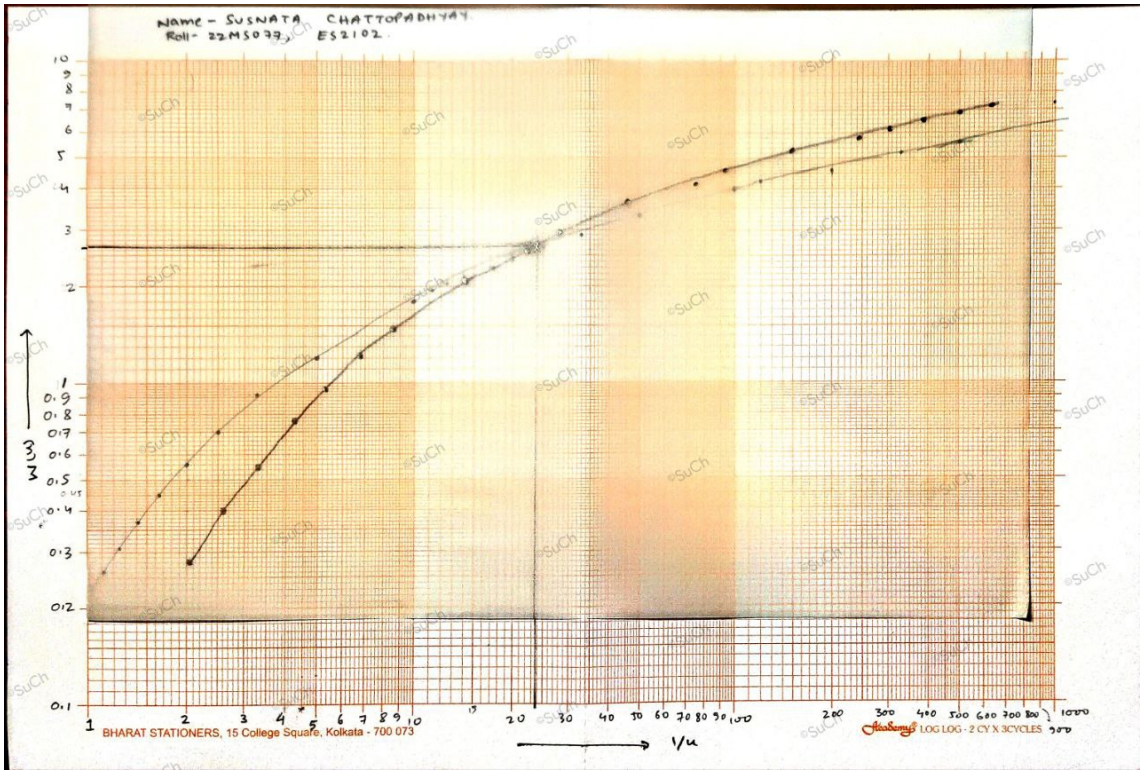


Figure 7: Overlay of tracing paper

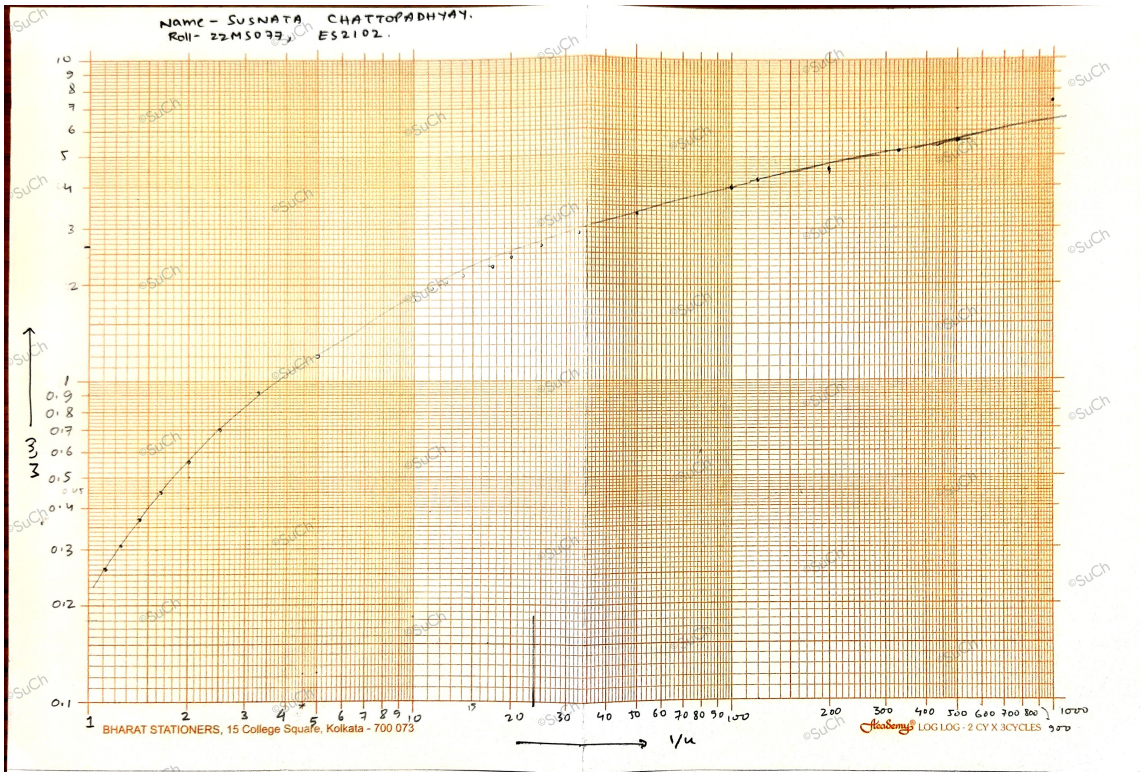


Figure 8: Plot of well function $(w(u))$ v/s $\frac{1}{u}$

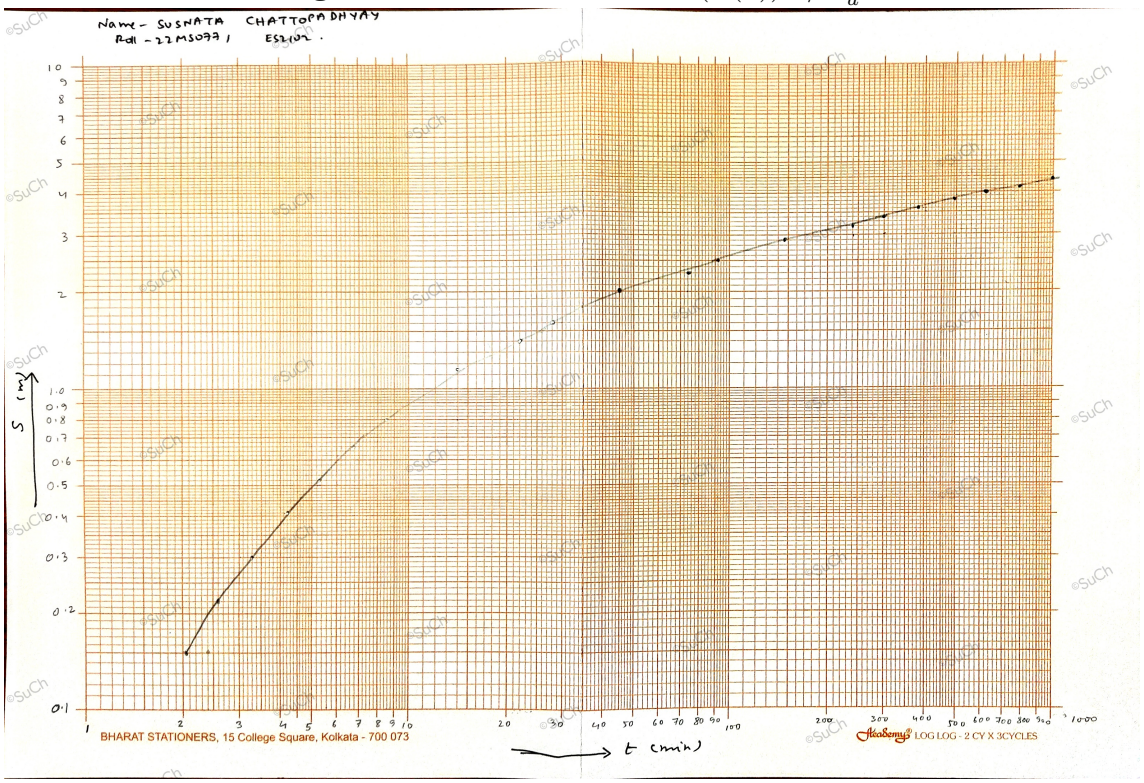


Figure 9: Plot of drawdown (s) v/s pumping time (t)

4 Week-1.4

`Read class notes of these topics once`

4.1 Determining residence time of water in aquifer- $^{14}_{6}C$ Method

The activity of $^{14}_{6}C$ and $^{3}_{1}H$ are measured to determine the age of groundwater in an aquifer. Until,the aquifer was getting recharged by external source which was connected to atmosphere, their concentrations remained constant. But as soon the source was disconnected, they start to decay. $^{14}_{6}C$ is produced in the upper atmosphere through Spallation Reaction, under the influence of *cosmic rays.*

Note that: $\frac{^{12}C}{^{14}C} = 10^{12}$

Spallation Reaction

It is a reaction in which a nucleus struck by a high energy particle shatters into two or more pieces, including stable and unstable nuclei, as well as protons and neutrons. The interaction of a cosmic ray with a nucleus sets of a chain reaction of sorts as the secondary particles and nuclear fragments, which themselves have very high energies, then strike other nuclei producing additional reactions of lower energy.

$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}p$

$^{14}_{6}C + O_2 \longrightarrow ^{1}_{4}CO_2$

This form of radioactive carbon is then consumed by plants and other organisms. It again decays by-

$^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{1}e^{-} + \nu(\text{anti})$

The activity is measured by mass spectrometer.

$$A_t = A_0 e^{-\lambda t}$$

The value of A_0 is obtained from atmosphere. So if the aquifer is determined as young, then it must be connected to atmosphere and is getting recharged. $t_{1/2} = 5730 \pm 40$ years. So, $\lambda = \ln 2/t_{1/2}$.

Note that: One of the primary requirements is that the aquifer must be disconnected from the atmosphere, before applying this method.

But there is a problem, the thickness of troposphere varies across latitudes, so $^{14}_{6}C$ production also varies, and so is A_0, across the globe. Moreover, the sunspot cycle also determines the cosmic ray production, which will decide the reaction. So, how to fix a standard value? This is done by a method illustrated below-

Dendrochronology A scientific method of dating *tree rings* (also called growth rings) to the exact year they were formed in a tree. The width of a tree ring reflects the amount of growth that occurred during that year. In most trees, growth is greatest in the spring and early summer, when there is plenty of sunlight and water. Growth slows down in the late summer and fall, and stops altogether in the winter. The width of the ring is therefore a measure of the climate conditions during that year.

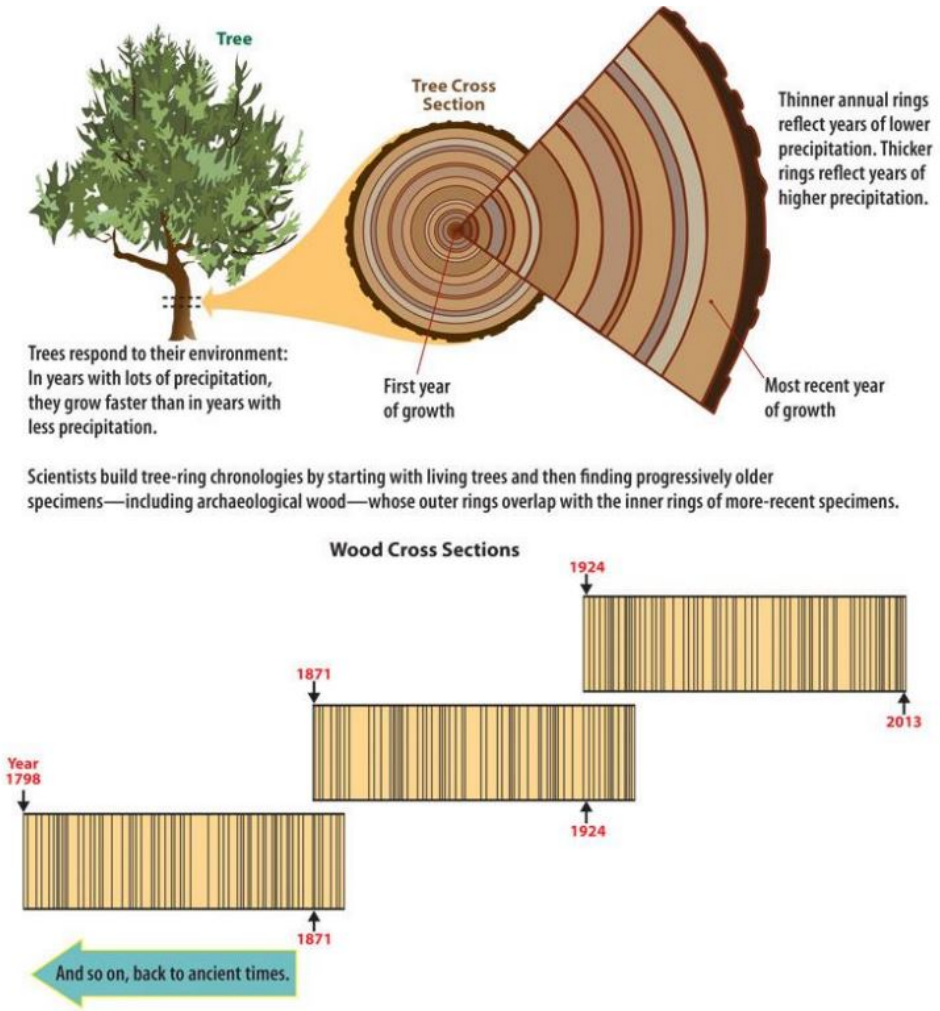


Figure 10: Dendrochronology

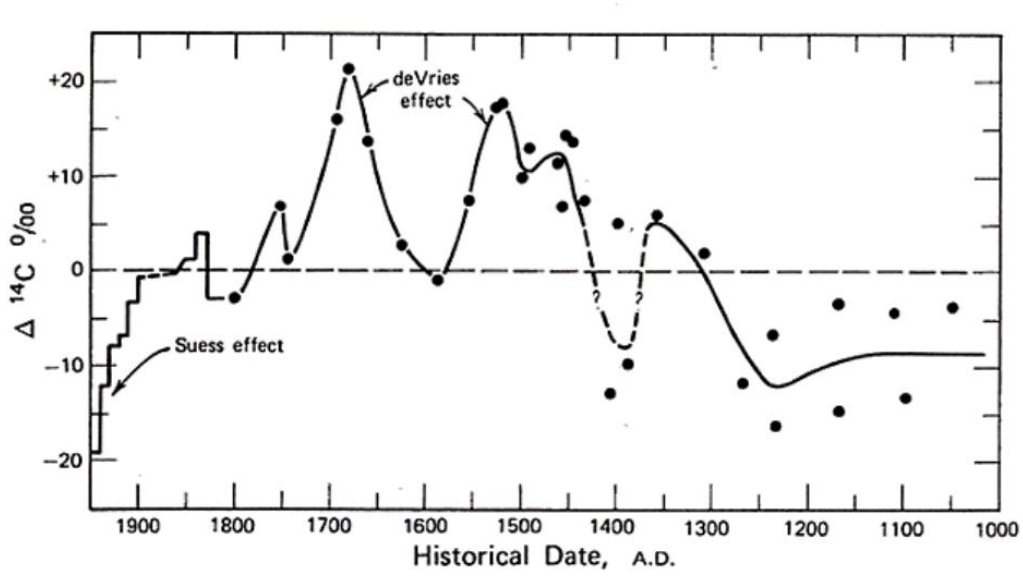


Figure 11: Deviation of initial radiocarbon activity in per mil of wood samples of known age

4.2 Factors affecting ^{14}C composition

Note that: The present is taken as 1950, since large amounts $^{14}_{6}C$ entered the atmosphere due to several thermonuclear bomb tests. **BP-** Before present.

deVries Effect It is the anomalous *increase* in the ratio of atmospheric $^{14}_{6}C$ to $^{12}_{6}C$ (See Fig 11), which *can* be caused due to changes in solar activity or Earth's magnetic field. When the Sun's activity is high, it produces more cosmic rays, which leads to an increase in the production of $^{14}_{6}C$. The Earth's magnetic field protects the Earth from harmful solar radiation. When the Earth's magnetic field weakens, it allows more solar radiation to reach the atmosphere, which can lead to an increase in the production of $^{14}_{6}C$.

Suess Effect It is the *decrease* in the ratio of atmospheric $^{14}_{6}C$ to $^{12}_{6}C$ due to the combustion of fossil fuels. Fossil fuels are made up of carbon that has been buried for millions of years. This carbon is depleted in $^{14}_{6}C$. So, when fossil fuels are burned, they release CO_2 into the atmosphere, which dilutes the concentration of $^{14}_{6}C$. This is known as the Suess effect.

4.3 $CaCO_3 - CO_2 - H_2O$ Equilibrium

$$H_2O + CO_2 \leftrightarrow H_2CO_3$$

$$H_2CO_3 \leftrightarrow H^+ + HCO_3^-$$

$$CO_3^{2-} + H^+ \leftrightarrow HCO_3^-$$

$$CaCO_3 \leftrightarrow Ca^{2+} + CO_3^{2-}$$

$$CaCO_3 + H_2O + CO_2 \leftrightarrow Ca^{2+} + 2HCO_3^-$$

This is also known as method of ppt of $CaCO_3$, of determining $^{14}_{6}C$ concentration. Here, we use only the CO_2 concentration, because it is purely from atmosphere, We do not take HCO_3^- or $CaCO_3$ as they dilute the concentration of $^{14}_{6}C$. Moreover $CaCO_3$ is present in older rocks, so much of the $^{14}_{6}C$ is decayed.

Problem 6 If we use HCO_3^- for measuring activity of $^{14}_{6}C$. How will our estimate of time be affected?

Solution: Let A be the true activity while, A' be the diluted activity measured.
$\therefore t = \frac{1}{\lambda} \ln \frac{A_0}{A}$ and, $t' = \frac{1}{\lambda} \ln \frac{A_0}{A'}$

Subtracting, $t - t' = \frac{1}{\lambda} \ln \frac{A'}{A} < 0 \ (\because A' < A) \implies t' > t.$

which means it is an **overestimation.**

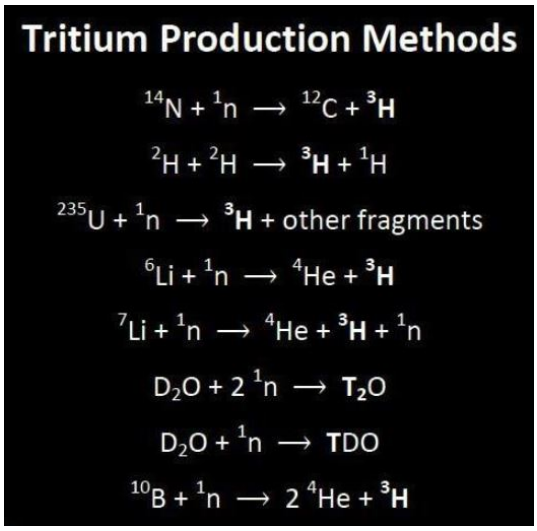
Problem 7 If ^{14}C is used to find out the age of groundwater in a limestone aquifer, what will be the uncertainty on the estimated age?

Solution: In this case most of ^{14}C present in limestone rocks is already *dead*. So their activity is diluted and from the above reasoning we can conclude it will result in an overestimation of age.

Note that: $^{14}_{6}$C is a long-lived isotope, so it cannot be used to measure recent changes in aquifer. It is used to find the **residence time**, which is a a measure of the average time a molecule of water spends in a reservoir. The residence time defined for steady-state systems is equal to the reservoir volume divided by the inflow or outflow rate.

5 Week-1.5

5.1 $^3_1$H method to determine infiltration rate of Aquifer



Tritium is a radioactive isotope of hydrogen with a half-life of *12.32 years*. It is produced in the atmosphere by the interaction of cosmic rays with nitrogen atoms.
$^{14}_7\mathrm{N} + {}^1_0\mathrm{n} \longrightarrow {}^{12}_6\mathrm{C} + {}^3_1\mathrm{H}$
$^3_1\mathrm{H} \longrightarrow {}^3_2\mathrm{He} + {}^0\beta_{-1}$

Tritium can be used to measure the age of aquifers because its concentration in precipitation has increased dramatically since the 1962 due to atmospheric nuclear weapons testing. This is known as the **tritium bomb pulse**.

Different depths of aquifer will contain different concentrations of $^3_1$H as they do not mix. It is used to measure the rate at which rainwater is infiltrating the aquifer. Presence of tritium implies recharge is still going on and aquifer is active, otherwise it is ded(hehe...). Tritium is incorporated into water molecules as they form in the atmosphere. This method of redistribution of isotopes within water molecules is called **fractionation.** It occurs through *exchange reaction*.
$\mathrm{T} + \mathrm{H_2O} \iff \mathrm{HTO} + \mathrm{H}$

Since, $^3_1$H is short lived we do not measure it's activity, instead we measure that of it's decay products, i.e $^3_2$He

$$\text{Infiltration rate} = \frac{\text{Depth}}{\text{time}}$$

$$D = N_0 - N = N(e^{\lambda t} - 1)$$
$$N_{^3\mathrm{He}} = N_{^3\mathrm{H}}(e^{\lambda t} - 1)$$

Aquifers that were recharged after the 1960s will contain some tritium, while aquifers that were recharged before the 1950s will not contain any tritium. By measuring the concentration of tritium in an aquifer, it is possible to determine when the aquifer was recharged. However, it is important to note that the tritium dating method is not without

its *limitations*. One limitation is that it can **only be used to** date aquifers that are relatively young, typically **less than 50** years old. Another limitation is that the method can be affected by contamination from other sources of tritium, such as nuclear accidents.

5.2 Arsenic Problem

Out of the various forms of Arsenic, As^{+3} is the one which causes us trouble. As per WHO it's concentration should be less than 10 ppb in drinking water.

5.2.1 How is As mobilised in water?

1. Arsenopyrites Arsenic is mainly found in river water sediments in adsorbed phase, in a mineral called *Arsenopyrite* (AsFeS). Due to excess withdrawal of water, these sediments are exposed to air and oxidized. As a result it releases As into water.
$2\,FeAsS + \frac{17}{2}\,O_2 + 4\,H^+ \longrightarrow Fe_2O_3 + 2\,H_2AsO_3^- + 2\,SO_4^-$. However it has been found in many places where As^{3+} conc. is high, Fe exists mainly in +2 state. So, this theory alone cannot be the major contributor to As^{3+}.

2. Excess Fertilizer use Chemical fertilizers contain lots of nitrates and phosphates. These often drain into nearby water bodies from field by soil runoff. Due to similar size of PO_4^{3-} it replaces $As(OH)_3$, which was locked in the sediments thus releasing it into water. Again, As conc. were found high in regions devoid of any agricultural activity.

3. Reduction of FeO(OH) This is the most plausible mechanism for the production of As^{3+} in water. Arsenic is adsorbed on Iron(III) oxide-hydroxide(FeO(OH)). Decomposition of organic matter releases acids, this reduces FeO(OH), and consequently releases As^{3+} in aquifer.
$FeO(OH) + CH_3COO^- + H_2CO_3 \longrightarrow Fe^{2+} + HCO_3 + H_2O$
Note that: It is said that while digging for tubewell, if we find *red/pink* sand then it's okay, but discontinue using that area if we find *white* sand. Why is it so? Pink sand has a coating of FeO(OH), which keeps As adsorbed. Due to the release of acids, the coating is reduced and thus the sand appears white/ grey, thus the corresponding aquifer is contaminated with Arsenic. One should also keep in mind that, a thick layer of clay will not allow organic acids to percolate into sand. Thus protecting it's layer.

5.2.2 Why are Deep Aquifers less contaminated compared to shallow aquifers?

Generally in Bengal basin, deep aquifers(lensoidal shape) are present under a thick layer of clay, so it is protected by the above reason. And secondly during *Last Glacial Maxima* (LGM), [snowball earth time..], M.S.L. dropped by 120m. This caused a huge hydraulic gradient. So, the heavy metals were flushed out of the continents into oceans, leaving behind pristine pure water in the aquifers.

6 Week-1.6

Groundwater Flow Modelling

Various Models

1. Sand Tank Model
2. Analog Model- Electrical Resistivity Method
3. Numerical Model
   (a) Finite difference Method
   (b) Finite element Method

6.1 Sand Tank Model

A sand tank model is a physical model of groundwater flow. It is a transparent tank filled with sand, with water added to the top of the tank. The sand is used to represent the aquifer, and the water represents the groundwater. The model can be used to study the flow of groundwater through an aquifer, the effects of different factors on groundwater flow, and the movement of contaminants through groundwater.

The sand tank model can be used to study a variety of factors that affect groundwater flow, such as:

- The permeability of the aquifer
- The slope of the land surface
- The presence of recharge areas
- The presence of pumping wells
- The movement of contaminants through groundwater

6.2 Analog Model

An analog model of groundwater flow is a physical model that uses electrical, mechanical, or hydraulic properties to simulate the flow of groundwater. The model is constructed to be similar in shape and dimensions to the actual aquifer system. The electrical properties of the model are used to represent the hydraulic conductivity of the aquifer, and the mechanical properties of the model are used to represent the storage coefficient of the aquifer.

See Fig 12, Consider a multiple fracture system within a hard rock through which water flows. Here, w = width of flow which is $\perp$ to the direction of flow of fluid. Some experimental deductions-
$Q \propto b^3$, and Permeability due to single fracture is, $K_S = \frac{\rho g b^2}{12\mu}$.

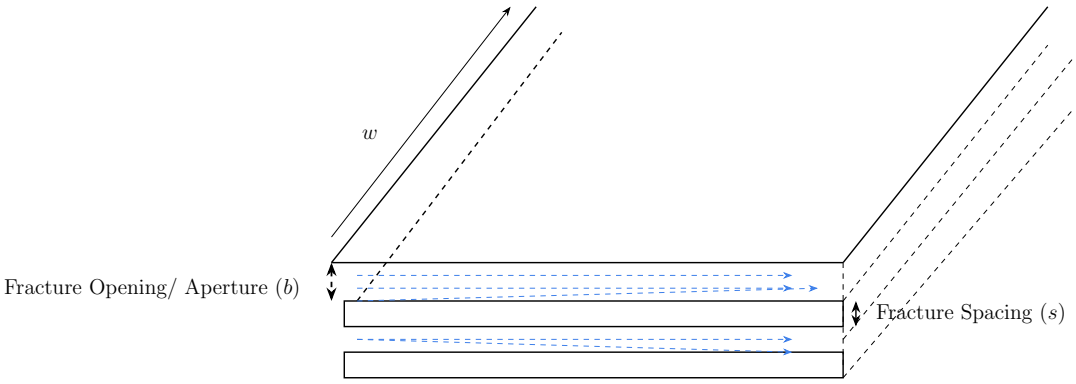


Figure 12: Model of Multiple Fracture System

Area of cross section, $A = bw$
Hence from Darcy's Law we get,

$$Q = K_S A \frac{\mathrm{d}h}{\mathrm{d}l} \implies Q = \frac{\rho g w}{12\mu} b^3 \frac{\mathrm{d}h}{\mathrm{d}l}$$

Now, in multiple fracture systems, we define **Fracture frequency** as, $N = 1/s$.
For loose sediment, porosity is η, but for hard rock with multiple fracture, it is defined as

$$\phi_f = \frac{b}{s} \left(\because \frac{bwx}{bwx + swx} \simeq \frac{bwx}{swx}, b << s \right)$$

Intrinsic permeability is defined as $k_{MF} = \frac{b^3}{12} N = \frac{K_{MF}\mu}{\rho g}$
So, we get Hydraulic conductivity as $K_{MF} = \frac{b^3 \rho g}{12\mu} N$.
Note that:

$$K_{MF} = \phi_f K_S$$

`[Proof: Easy]`
Typical value of viscosity of water in most aquifers is, $\mu = 1.002 \times 10^{-3}$ kg-s/m

6.3 Resistivity Method

The resistivity method is a geophysical method that uses the electrical resistivity of subsurface materials to map the subsurface. The resistivity of a material is its ability to resist the flow of electrical current. Groundwater is typically more conductive than the surrounding rock, so the resistivity method can be used to identify the presence of groundwater. Moreover, clay has lower resisistivity than sand (due to cations), thus the variation of *apparent resistivity*(ρ_a) with depth, can be used to model subsurface layers.

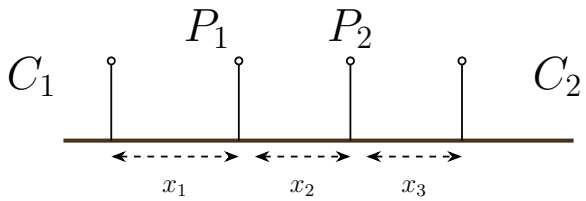


Figure 13: Configuration of Current electrode(C_i) and Potential electrode(P_j)

Note that: $[r_{ij}]$ denotes the distance between C_i and P_j. i.e r_{12} is the distance between C_1, P_2.

Following the laplace equation in spherical coordinates, we find- (Why is there no dependency on θ, ϕ ??)

$$\begin{aligned}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) &= 0\\ \Rightarrow r^2\frac{\mathrm{d}V}{\mathrm{d}r} &= c_1\\ \Rightarrow V &= \int_\infty^r \frac{c_1}{r^2}\mathrm{d}r\\ \Rightarrow V &= -\frac{c_1}{r}\end{aligned}$$

Now, we know that $E = -\frac{\partial V}{\partial r} = \rho j$. $\therefore c_1 = -r^2\rho j$. Hence we get,

$$V(r) = \frac{I\rho}{2\pi r}$$

The current is spreading *hemispherically* isotropically from C_1 point. So $j = I/2\pi r^2$. The readings of the voltmeters are-

$V_1 = \frac{I\rho}{2\pi}\left(\frac{1}{r_{11}} - \frac{1}{r_{21}}\right)$

$V_2 = \frac{I\rho}{2\pi}\left(\frac{1}{r_{12}} - \frac{1}{r_{22}}\right)$

So, the potential drop is

$$\Delta V \equiv V_1 - V_2 = \frac{I\rho}{2\pi}\left(\frac{1}{r_{11}} - \frac{1}{r_{21}} - \frac{1}{r_{12}} + \frac{1}{r_{22}}\right) = \frac{I\rho}{2\pi\kappa}$$

κ is the array constant or **geometrical factor** of electrode configuration.

1. **Wemer Configuration:**
   Electrodes are symmetric with respect to each other. $(x_1 = x_2 = x_3 = a = \kappa)$

   $$\rho_{app} = 2\pi\frac{\Delta V}{I}\kappa$$

2. **Schlumberger Configuration:**
   Here, four electrodes are placed in line around a common midpoint $(x_2 = a,\ x_1 = x_3 = s)$. Apparent resistivity is measured as,

   $$\rho_a = \frac{\pi(s^2 - a^2/4)}{a}\frac{\Delta V}{I}$$

Limitations of this model

- *Resolution*- Let, d, be the depth of the layer we are measuring and a be its thickness. Then this method works only if $\frac{a}{d} \geq 0.1$.

- *Suppression*- Suppose there are multiple layers with contrasting resistivities, and we are measuring a thick layer with high resistivity, and below it there exists a thin layer with comparatively lesser ρ_a, then this layer will **not** be detected, as the preceding layers have dominated over it's reading.

- *Equivalence*- Problem arises if layers have equivalent resistivities, even though they have different composition.

Note that: The best place to dig a bore-well is where we can find a buried channel (made of sand–*white*), this is identified by a gradual smooth bump on the graph of apparent resistivity with depth. Clay, feldsparic (Pink) Sand have low resistivity.

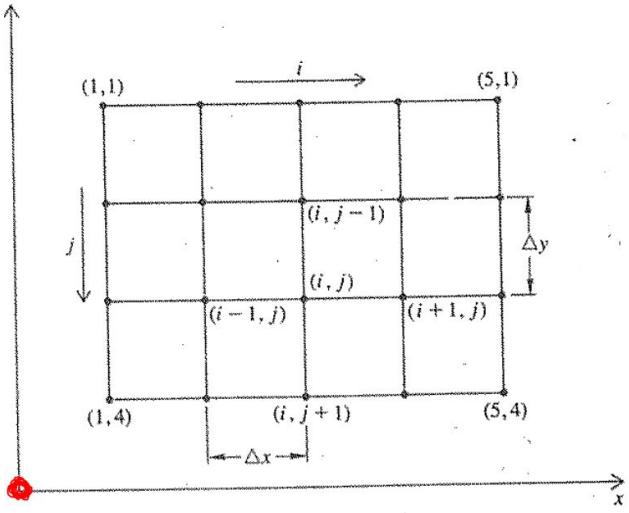


Figure 14: Break your area of investigation into grids as shown. The origin, in red marks the location of pumping well

6.4 Finite difference Method

Here we have to determine the hydraulic head values at each node, named as $[h_{ij}]$ according to the convention given in (Fig 14). We are working under the assumption that the aquifer is **homogeneous**.
Consider a 2-D steady state flow, in a confined aquifer;

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

From the use of taylor expansion we get,

$$\frac{\partial^2 h}{\partial x^2} \simeq \frac{h_{(i+1,j)} + h_{(i-1,j)} - 2h_{(i,j)}}{(\Delta x)^2}$$
$$\frac{\partial^2 h}{\partial y^2} \simeq \frac{h_{(i,j+1)} + h_{(i,j-1)} - 2h_{(i,j)}}{(\Delta y)^2}$$

If the grids are square, we have, $\Delta x = \Delta y$, then putting in laplace's equation we obtain-

$$4h_{(i,j)} = h_{(i+1,j)} + h_{(i-1,j)} + h_{(i,j+1)} + h_{(i,j-1)}$$

We use the above equation to solve for the unknowns. Note, the area should be well inside the radius of *cone of depression.*

7 Exercises

Problem 8 Ground water flows through a buried-valley aquifer with a cross sectional area of 1.0×10^6 ft^2 and a length of 2×10^4 ft. Hydraulic heads at the ground water entry and exit points in the aquifer are 1000 and 960 ft respectively. At the downstream end of the aquifer, ground water discharges into a stream at a rate of 1.0×10^5 ft^3 /day. What is the hydraulic conductivity of the buried-valley aquifer?

Problem 9 A well penetrates an unconfined aquifer. Before pumping, the water table is in $h_0 = 25$ m. After a long period of pumping at a constant rate of 0.5m ^{3} /s, the drawdowns at distances of 50 m from the well were observed 3 m. If $K = 7.26 \times 10^{-2}$ m/s find the drawdowns at distances of 50 m from the well.

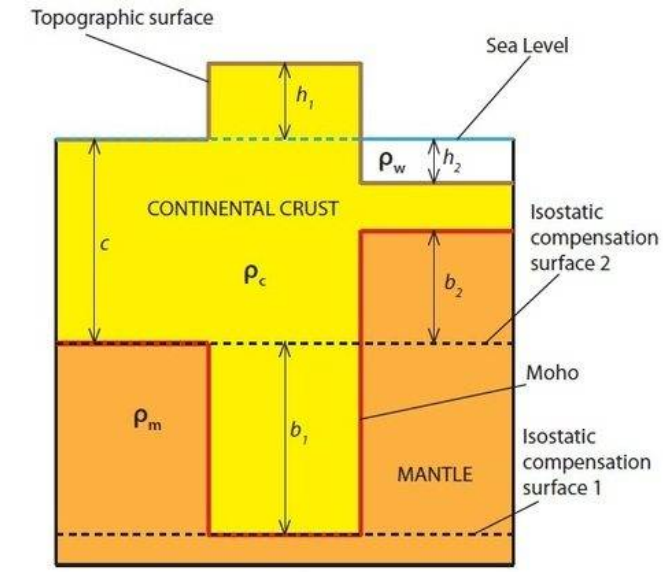
Module 2: Geomorphology and Geodynamics

8 Week-2.1

8.1 Determination of root of any elevated topography

We calculate this by understanding the *principle of isostacy*. Also known as isostatic equilibrium or isostatic adjustment describes the balance between the Earth's lithosphere (the rigid outer shell) and the underlying asthenosphere (the semi-fluid layer). It primarily applies to the continental crust but also has implications for the oceanic crust.

- The Earth's lithosphere "floats" on the denser, partially molten asthenosphere beneath it, much like an *iceberg* floats on water. The lithosphere is in a state of buoyant equilibrium, with thicker and denser parts of the crust rising higher in the mantle, and thinner and less dense parts sinking lower.
- So, as the topography is elevated by deposition of materials, some part of it (it's root) will also have to sink deeper into the asthenospheric mantle to a depth called *depth of compensation* (d_{cm}). It is at this depth, we encounter density contrast. Hence high mountain belts must have a deep root projected into the asthenosphere.



h_1 = elevation of mountain belt (above sea level)
h_2 = depth of marine basin (below sea level)
b_1 = thickness of crustal roots (below depth of Moho in a cratonic area)
b_2 = thickness of lithosphere mantle bulge (above depth of Moho in a cratonic area)
c = thickness of continental crust in an undeformed (cratonic) area (ca. 35 km)

ρ_w = density of sea water (ca. 1,000 Kg/m^3)
ρ_c = density of continental crust (ca. 2,800 Kg/m^3)
ρ_m = density of mantle (ca. 3,300 Kg/m^3)

Figure 15: b_1 is the root while b_2 is called *antiroot*

8.1.1 Calculation of root size

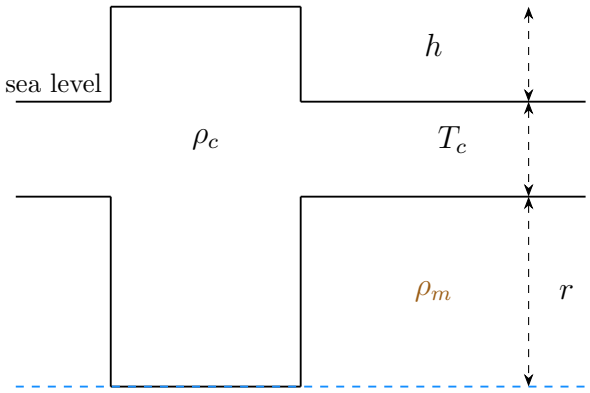


Figure 16: Here, h is elevation of the mountain/ plateau. T_c is average crustal thickness. r= root and the dashed- *blue* line denotes the depth of compensation.

At d_{cm}, we have pressures equal. So, it follows that

$$(h + T_c + r)\rho_c g = \rho_c T_c g + \rho_m r g$$

$$\boxed{r = \frac{h\rho_c}{\rho_m - \rho_c}} \tag{1}$$

The total thickness of the crust is $T = h + T_c + r$. Conversely for a basin of depth $d = -h$, (equal to height h) it's **antiroot** will be of size $\bar{r} = r$. The mean elevation (h) of the land surface above sea level is called the *continental freeboard*.

Note that: The thickness of the crust (T_c) is calculated using seismic observations near coastal regions. It's value is $\simeq 35$ km.

8.2 Erosion of Mountains

The rate of erosion of mountains/ elevated topography, is given by

$$E = k(\Delta P)^m \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^n$$

where, ΔP is the rate of precipitation (higher in frontal parts of mountain) and $\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)$ is the gradient of topography.

Observe that to erode a plateau to sea level, one must remove the part $(r + h)$. Because as one tries to decrease h, due to buoyancy it will again rise up as $r \downarrow$. So the total removed part is essentially $h + r$.

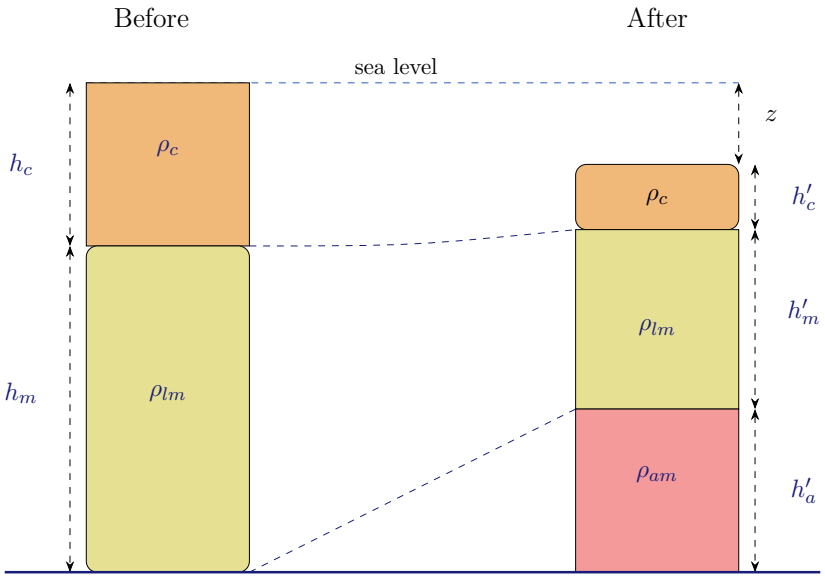
Typical values

- Crust, $\rho_c = 2800$ kg/m^3
- Lithospheric Mantle, $\rho_{lm} = 3300$ kg/m^3
- Asthenospheric Mantle. $\rho_{am} = 3250 - 2830$ kg/m^3

8.3 Pratt-Hayford Isostasy Model

Here, the density of the crust above sea level is the same as the density of the crust at the coast. While the density of the crust below sea level varies laterally, being less under mountains than under oceans.

Consider a continental Lithosphere comprised of 30km thick crust, and 90 km thick mantle. After an episode of continental extension, the thickness of lithosphere became 1/2 the original thickness. This thinning of crust beneath the ocean is known as *anti-root*. Calculate the depth(z) of the resulting sedimentary basin, if it's filled with water(ρ_w) / sediment(ρ_s) respectively.



Given, $h_c = 30$ km, $h_m = 45$ km, $h'_c = 15$ km, $h'_m = h_m/2$ km, and z, h'_a =?. Observe that by equating heights below sea level,

$$h'_a = (h_c - h'_c) + (h_m - h'_m) - z$$

and writing pressure/g, at depth of compensation.

$$\rho_c h_c + \rho_{lm} h_m = \rho z + \rho_c h'_c + \rho_{lm} h'_m + \rho_{am} h'_a$$

Substituting values and solving we get,

$$z = \frac{1}{\rho - \rho_{am}}((h_c - h'_c)(\rho_c - \rho_{am}) + (h_m - h'_m)(\rho_{lm} - \rho_{am}))$$

For, $\rho = \rho_w$, one gets $z \simeq 1.3$ km. And for, $\rho = \rho_s = 2.3$ g/cc, one gets $z = 3$ km.

8.4 Airy–Heiskanen Isostacy Model

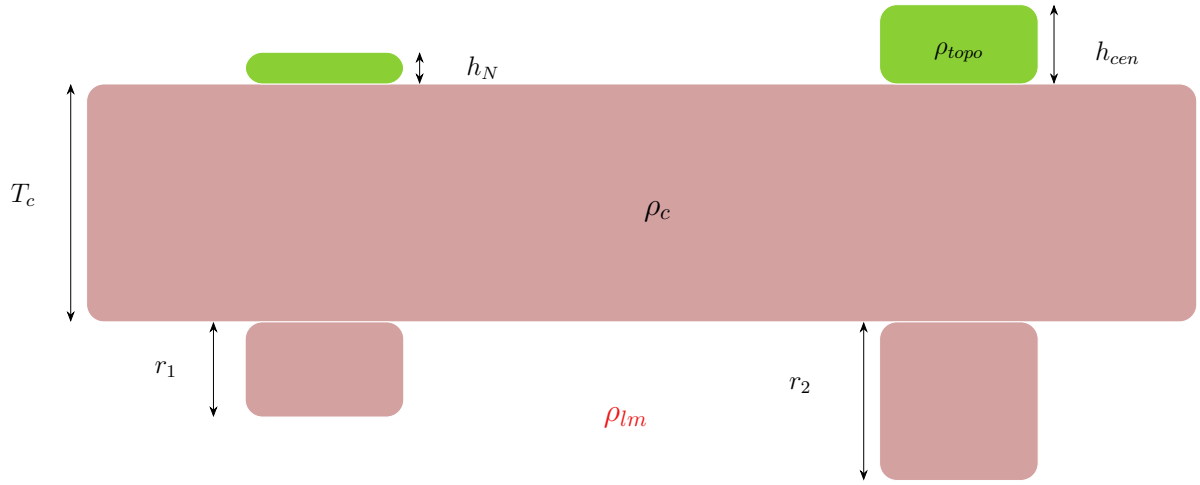


Figure 17: This diagram depicts the tomography of the Alps. The space between the roots is the lithospheric mantle ρ_{lm}.

Here we have a tomographic model of northern and central Alps. The lithospheric mantle (yellow in diagram) begins at the Moho discontinuity. We need to determine the elevation of the mountains above sea level, (i.e h_N and h_{cen}). Balancing pressure at depth of compensation below each mountain range we get,

$$h_N\rho_{topo} + (T_c + r_1)\rho_c = T_c\rho_c + r_1\rho_{lm}$$
$$h_{cen}\rho_{topo} + (T_c + r_2)\rho_c = T_c\rho_c + r_2\rho_{lm}$$

However, it has been found that the observed elevations of these mountain ranges are *less* than the calculated heights. This can be due to a variety of reasons like the crust might not be of same thickness everywhere, we assumed a much lighter lithosphere, density values might differ.

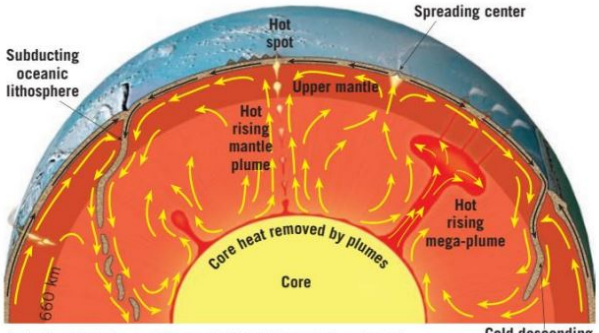
9 Week 2.2

9.1 Lithospheric Mantle Slab

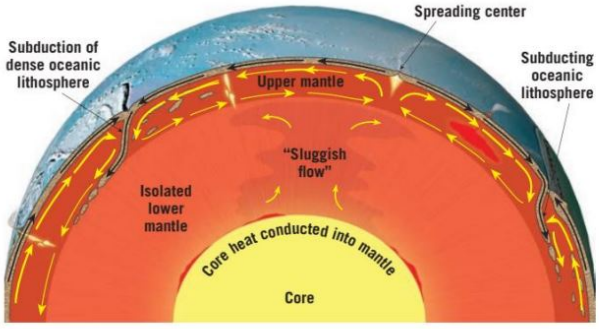
When two tectonic plates collide, one of the plates may be forced beneath the other. This process is called subduction. The subducting plate is typically denser than the overlying plate, so it sinks into the asthenosphere. As the subducting plate sinks, it heats up and releases water and other fluids. These fluids cause partial melting of the surrounding mantle, which leads to the formation of magma. These melting causes a region of *low velocity* zone which is visible from seismic tomographical observations. This section of the lithosphere that has been subducted beneath another tectonic plate, is called Lithospheric Mantle Slab (LMS).

Subducting slabs can penetrate deep into the mantle, reaching depths of up to 700 kilometers. As the slab sinks, it is slowly deformed and heated. The heat causes the minerals in the slab to transform into new minerals that are more stable at high temperatures and pressures. The slab also loses water and other fluids, which makes it more dense.

Over time, the subducting slab can reach a depth where it is so dense that it can no longer sink any further. At this point, the slab stalls and begins to accumulate beneath the overlying plate. This accumulation of subducted slabs is called the **slab graveyard**.

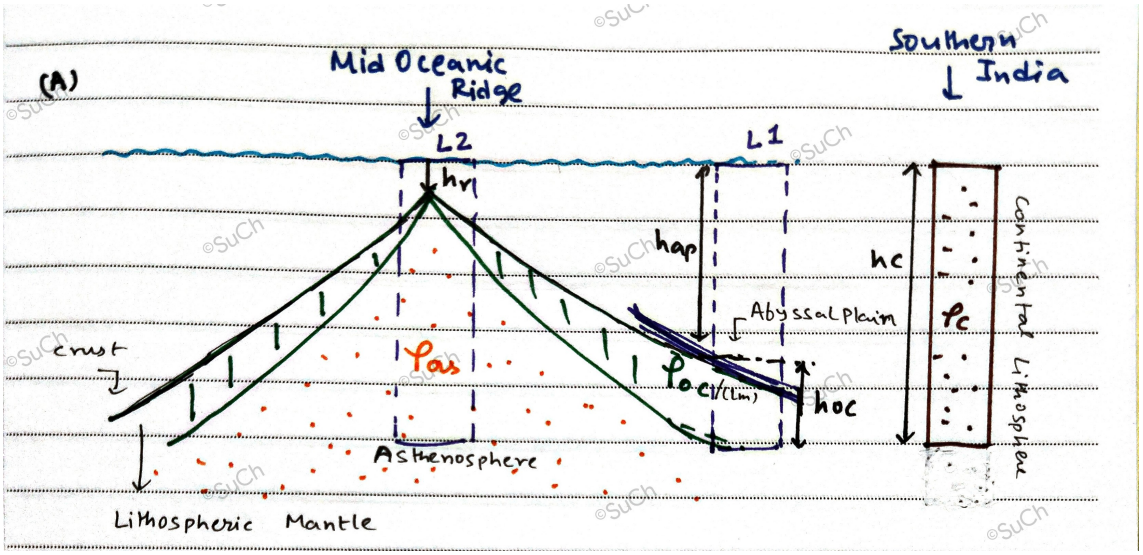


A. In the "whole-mantle model," sinking slabs of cold oceanic lithosphere are the downward limbs of convection cells, while rising mantle plumes carry hot material from the core–mantle boundary toward the surface.



B. The "layer cake model" has two largely disconnected convective layers. A dynamic upper layer driven by descending slabs of cold oceanic lithosphere and a sluggish lower layer that carries heat upward without appreciably mixing with the layer above.

9.2 Topography of Ocean Basin



Consider the Mid Oceanic ridge in the Indian ocean which is formed by a divergent plate boundary. We want to determine the depth of oceanic crust beneath the ocean floor. We take a reference coloumn at the base of Southern India, with which we shall compare with 2 locations.

Location 1

$$\boxed{h_{oc} = h_c \frac{\rho_c - \rho_w}{\rho_{oc} - \rho_w}}$$

Location 2 At the boundary, $h_{oc} = 0$. So we shall calculate the depth of opening.

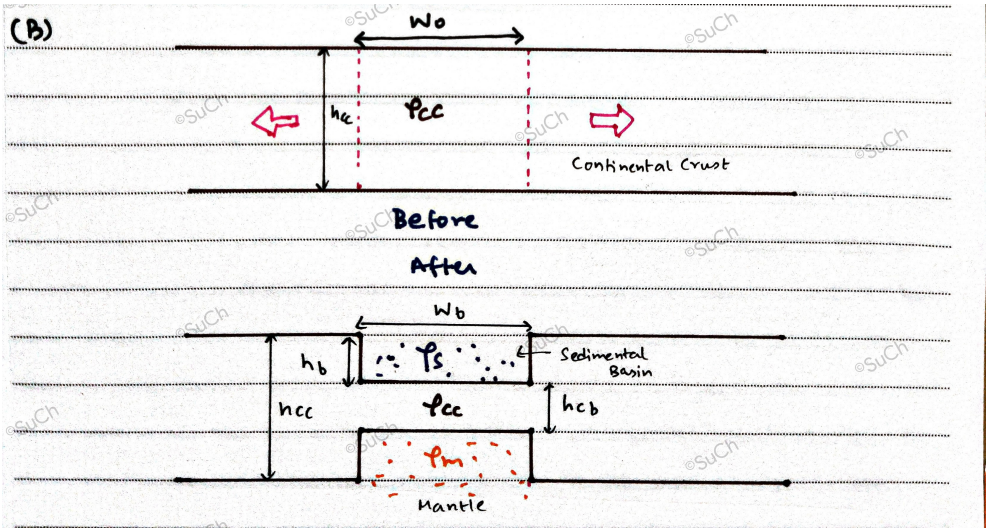
$$\boxed{h_r = h_c \frac{\rho_c - \rho_{as}}{\rho_w - \rho_{as}}}$$

Note that: `Derive them yourself!`

10 Week 2.3

In both the estimation, problem we assumed the volume remained constant, after the processes.

10.1 Estimation of Extensional Setting



Here, we define a quantity called **crustal stretching factor** (α). It is a measure of how much the crust has been thinned due to extension. It is defined as the ratio of stretched lithosphere width (or the width of basin formed by subsidence) (w_b) and width of original lithosphere (w_0).

$$\alpha = \frac{w_b}{w_0}$$

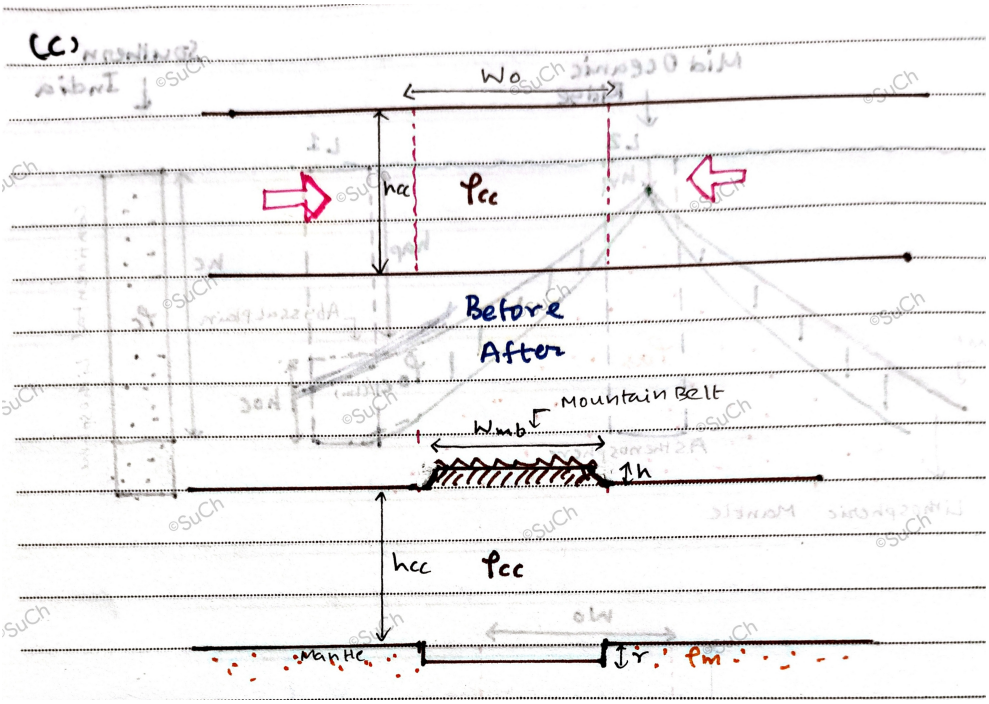
Conserving volume, and equating pressures we have the equations:

$$h_{cc}w_o = h_{cb}w_b$$
$$\rho_{cc}h_{cc} = \rho_s h_b + \rho_{cc}h_{cb} + \rho_m(h_{cc} - h_b - h_{cb})$$

Solving the 3 equations we obtain,

$$\boxed{\frac{1}{\alpha} = 1 - \frac{h_b(\rho_m - \rho_s)}{h_{cc}(\rho_m - \rho_{cc})}}$$

10.2 Estimation of Contractional Setting



Similarly, we define **crustal compression factor**(subscript mb denotes mountain belt/ plateau).

$$\beta = \frac{w_o}{w_{mb}}$$

Conserving volume, and equating pressures we have the equations:

$$h_{cc} w_o = (h + h_{cc} + r) w_{mb}$$
$$\rho_{cc} h_{cc} + \rho_m r = (h + h_{cc} + r)\rho_{cc}$$

Solving the 3 equations we obtain,

$$\boxed{\beta = 1 + \frac{h\rho_m}{h_{cc}(\rho_m - \rho_{cc})}}$$

11 Week 2.4

11.1 Gravity Anomaly and Corrections

We can identify the subsurface structure/ material by analysing the gravity anomaly measured by gravimeters set up at various positions on topography. At the sea level, the value of gravity varies across latitude because of rotation of earth. Thus the theoretical value of gravity at any latitude λ is given by

$$g_t(\lambda) = g_e[1 + \alpha \sin^2 \lambda + \beta \sin^4 \lambda]$$

where the value of gravity at equator $g_e = 978031.85$ mGal, $\alpha = 5.278895 \times 10^{-3}$, $\beta = 2.3462 \times 10^{-5}$.

11.1.1 Free Air Anomaly

Now consider we are in some isolatitude region. Then at sea level, gravity at each point are expected to be equal. However if we are at an elevation h above sea level due to topography, then naturally the gravity is expected to decrease(FAC = -ve). Conversely if we are at depth $(-d)$ the gravity must be more as compared to surface (FAC = +ve). This is called **free air correction** (FAC).

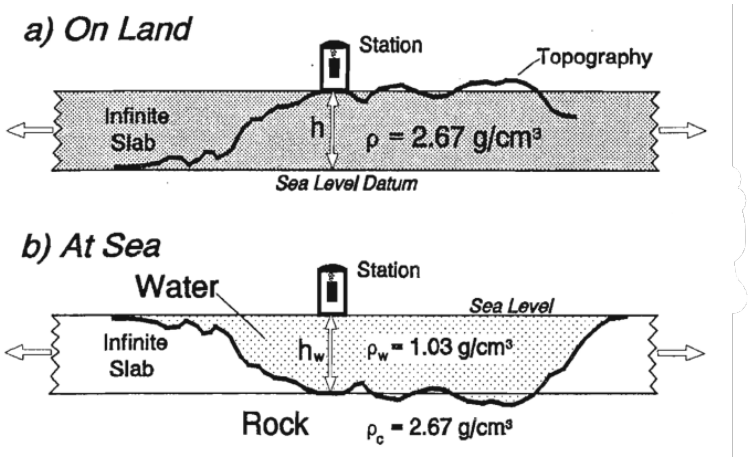
$$\text{FAC} \equiv -\frac{\mathrm{d}g}{\mathrm{d}R}\Delta h = -\frac{2g}{R}\Delta h = -0.308\Delta h \text{ mGal}$$

Correspondingly *free air anomaly* is

$$\Delta g_{fa} = g_{\text{obs}} - (g_t(\lambda) + \text{FAC})$$

At short wavelengths free-air gravity anomalies correlate strongly with local topography. To remove the gravitational attraction of the local topography, the Bouguer gravity formula is used.

11.1.2 Bouguer Anomaly



A) On Land If we are standing on top a mountain then from FAC gravity is expected to decrease, however we need to also take into account the extra mass due to topography (of the mountain) beneath our feet(gravimeter) and sea level. Conversely if we are in a valley, the void needs to be taken into account as gravity just above the valley at *sea level* will be less than normal. (Pause and Think). This is called **Bouger Correction** (BC).

$$\text{BC} \equiv 2\pi\rho G h = 0.0419\rho_{topo}(g/cc) \times h(m) \text{ mGal}$$

Correspondingly *bouguer anomaly* is

$$\Delta g_{ba} = g_{\text{obs}} - (g_t(\lambda) + \text{FAC} + \text{BC}) = \Delta g_{fa} - \text{BC}$$

A negative value of Δg_{ba} suggests that the mountain gravity is supported by lighter density crustal root. Thus the Bouguer gravity profile more or less portrays the geometry of subducted plate. Because the Bouguer gravity correction for topography does not account for this negative root, the anomalies over mountain ranges are strongly negative.

B) At Sea Only here, we just fill the basin with water. Thus as the crust is more denser than sea water, this means that BC will be negative.

$$\text{BC}_s = 0.0419(\rho_w - \rho_c)h_w$$

Note that: At the sea level FAC = 0 mGal.

11.1.3 Anomalous Mass

Now suppose, we encounter a subsurface lateral density contrast. This can be due to an anomalous mass of different density other than surrounding crust(ρ_c) lurking inside. Depending upon it's density(ρ_s) whether lighter or denser, it shall contribute ($\Delta\rho = \rho_m - \rho_c$) negatively or positively to the gravity correction. By Forward Gravity modelling we can approximate as a spherical body (Fig 18).

$$C_{anom} = 0.02794 \times \Delta\rho R^3 \frac{z}{(x^2 + z^2)^{3/2}}$$

where x, z is the horizontal and vertical distance(depth) of the mass respectively with respect to gravimeter.

Hence the total gravity at each point is given by

$$\boxed{g = g_t(\lambda) + \text{FAC} + \text{BC} + C_{anom}}$$

11.2 Gravity Modeling

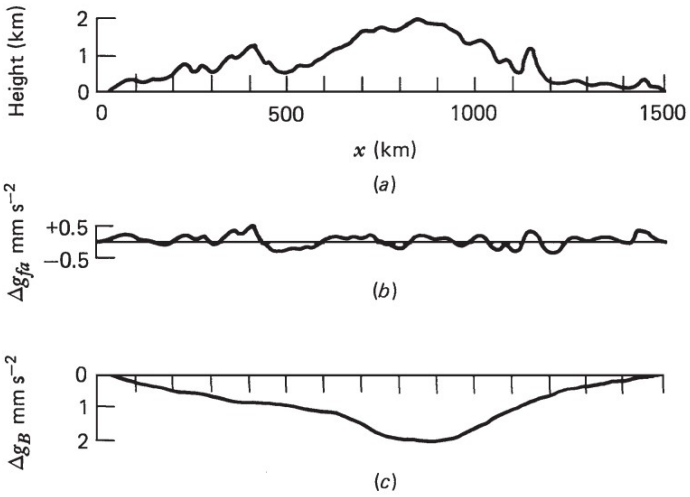


Figure Free-air (*b*) and Bouguer (*c*) gravity anomalies associated with the topography given in (*a*).

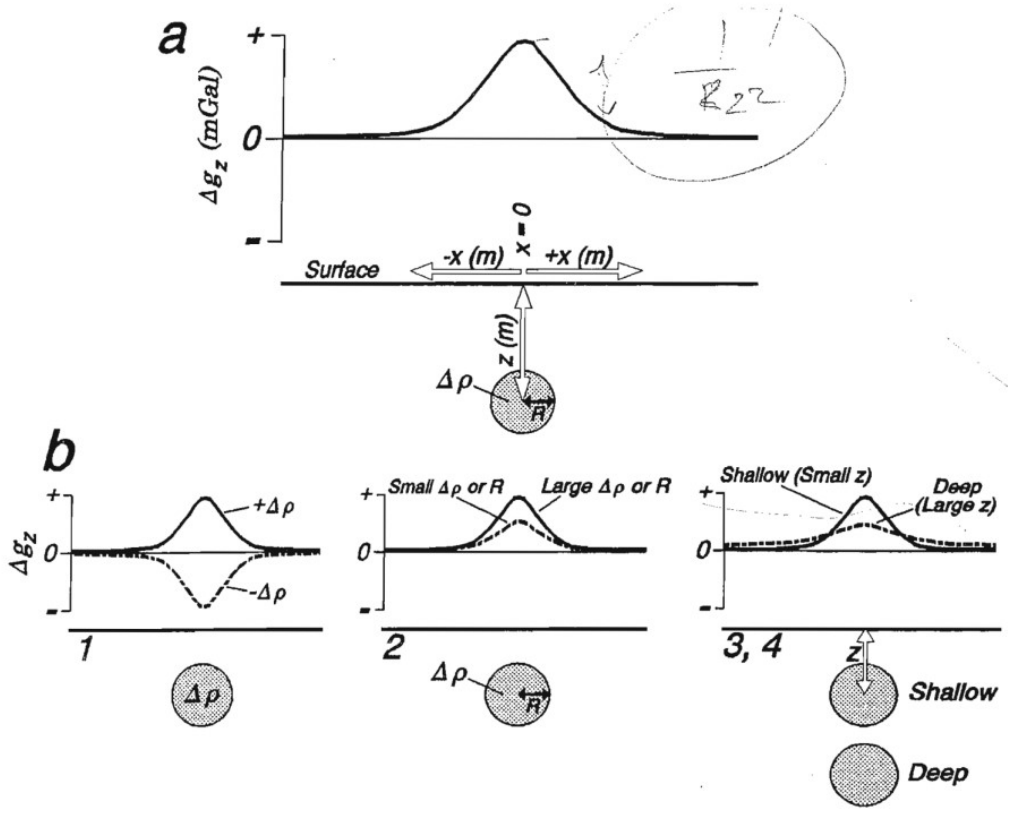
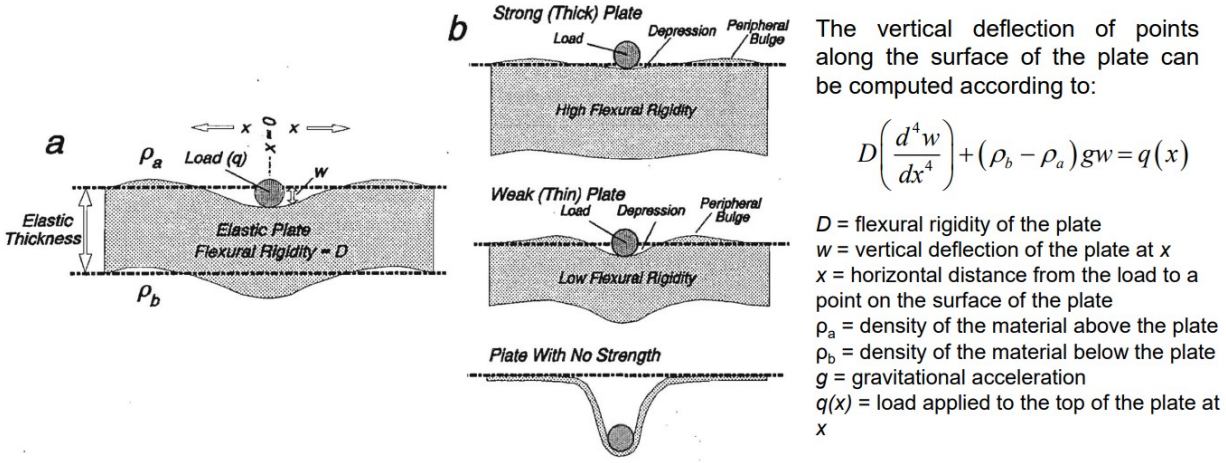


Figure 18: Effect of gravity profile by an anomalous mass.

12 Week 2.5

Lithospheric Flexure

Till now, we have worked with Local Isostacy which assumes the plate has no rigidity. In reality however the plates have a flexibility which enables it to bend under load (forming a foreland area), at the convergent mountain ranges (*Himalayas*). This is called **Regional Isostacy** as the topographic load is distributed over a larger area.



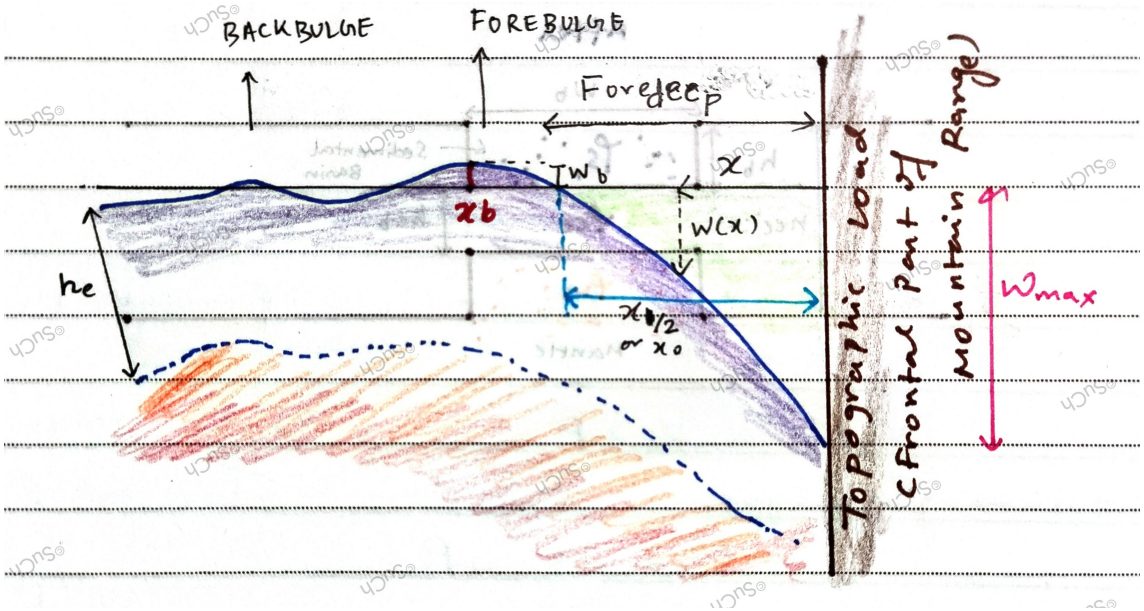


Figure 19: Foreland Basin of a mountain range

Foredeep is that part of the deforming plate over which sediment accumulates. ρ_s is the density of infilling unconsolidated sediments. h_e is called the elastic thickness of the plate. It is the geological thickness over which the plate behaves elastically. α is **flexural parameter** of the plate. D is **flexural rigidity**, it determines the ease with which the plate moves. $w(x)$ is the deflection at any point from the foot of the mountain. x_o or $x_{1/2}$ is the **half-width** of the plate. Y and σ are the usual Young's Modulus and poisson

ratio respectively. ρ_s is the density of infilling unconsolidated sediments. To locate the position of the forebulge (x_b) and half-widths we consider 2 cases.

Note that:

$$\alpha = \left[\frac{4D}{(\rho_{lm} - \rho_s)g}\right]^{1/4}$$
$$D = \frac{Yh_e^3}{12(1-\sigma^2)}$$

12.1 Plate is continuous

Here

$$w(x) = w_{\max} e^{-x/\alpha}[\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}]$$

At half width $w(x_{1/2}) = 0$. So,

$$x_{1/2}/\alpha = \tan^{-1}(-1) \implies \boxed{x_{1/2} = \frac{3\pi\alpha}{4}}$$

Again at central part of forebulge, slope is zero. i.e, $\frac{\mathrm{d}w(x)}{\mathrm{d}x}\Big|_{x_b} = 0$. Solving,

$$x_b/\alpha = \sin^{-1}(0) \implies \boxed{x_b = \pi\alpha}$$

Height of forebulge is then $w(x_b) = -0.0432 w_{\max}$

12.2 Plate is broken

Here the equation becomes

$$w(x) = w_{\max} e^{-x/\alpha}[\cos\frac{x}{\alpha}]$$

At half width $w(x_{1/2}) = 0$. So,

$$x_{1/2}/\alpha = \cos^{-1}(0) \implies \boxed{x_{1/2} = \frac{\pi\alpha}{2}}$$

Again at central part of forebulge, slope is zero. i.e, $\frac{\mathrm{d}w(x)}{\mathrm{d}x}\Big|_{x_b} = 0$. Solving,

$$x_b/\alpha = \tan^{-1}(-1) \implies \boxed{x_b = \frac{3\pi\alpha}{4}}$$

Height of forebulge is then $w(x_b) = -0.067 w_{\max}$

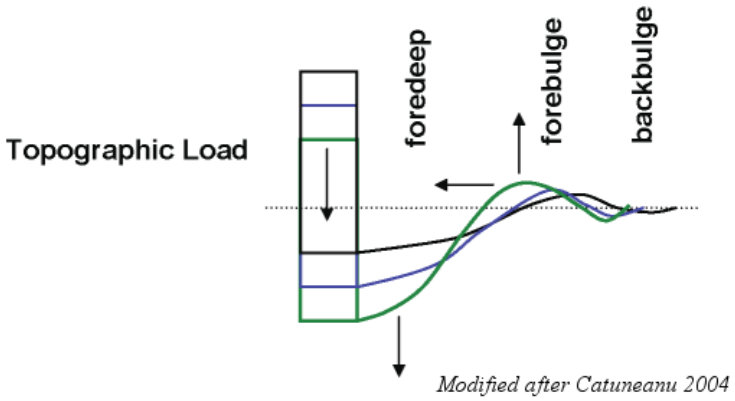
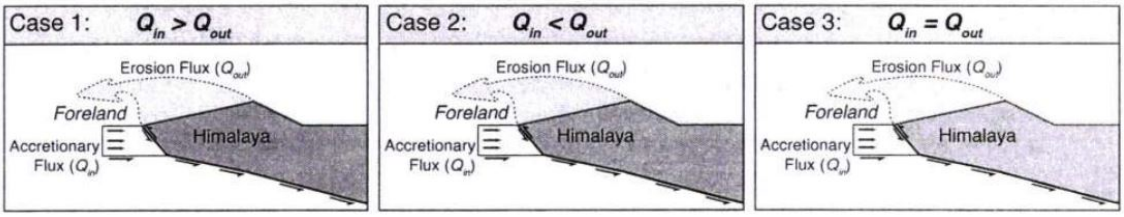


Figure 20: Moving Load System- Lithospheric flexure over time



Problem 10 Among the following three cases, which can explain the *forelandward growth* of the Himalayas?

Solution: We know as topographic load increases foreland basin decreases in area. So, foreland growth implies erosion flux must be greater than accretionary flux which results in more sediments being accumulated in the basin thus contributing to the growth of foreland. So **Case 2** seems the most viable.

13 Week 2.6

13.1 Deviatoric Stress

From this quantity ($\Delta\sigma_{XX}$) we get to know the balance the crustal converging forces ($\mathbf{F_C}$) and the force exerted by asthenospheric wind in the mantle ($\mathbf{F_m}$). Note that we are calculating the force *per unit length* of the crust. A *thickened* crust is generally formed at convergence zone or regions of uplift like mountains. Let the thickness of this crust (ρ_c) be h and the length of root protruding in the mantle (ρ_m) be b.

$$F_m = \int_0^b \rho_m g y \, \mathrm{d}y$$
$$F_c = \int_0^h \rho_c g y \, \mathrm{d}y$$

For the crust to sustain:

$$F_c + \Delta\sigma_{XX} \times h = F_m$$

After isostatic calculations we find

$$\Delta\sigma_{XX} = \frac{1}{2}\rho_c g h \left(\frac{\rho_c}{\rho_m} - 1\right)$$

Thus if $\Delta\sigma_{XX} > 0$ this implies $F_c < F_m$ and the crust will suffer contraction. Conversely, $\Delta\sigma_{XX} < 0$ implies $F_c > F_m$ and the crust will suffer extension.

13.2 Passive Margin

A passive margin is the transition zone between continental and oceanic lithosphere where there is minimal or no tectonic activity. It's the opposite of an active margin, which is characterized by intense tectonic activity like subduction or collision. F_c can be calculated in a similar way as stated above, here we shall look at how to calculate F_m. From the sea level we go down a distance y, then the corresponding pressure regions are given by

$$\begin{aligned} P(y) &= \rho_w g y && 0 < y \leq h_w \\ &= \rho_w g h_w + \rho_{oc} g y && h_w < y \leq h_w + h_{oc} \\ &= \rho_w g h_w + \rho_{oc} g h_{oc} + \rho_m g (y - h_w - h_{oc}) && h_w + h_{oc} < y \leq h_{cc} \end{aligned}$$

$$F_M = \int P(y) \mathrm{d}y$$

Try to deduce the diagram!