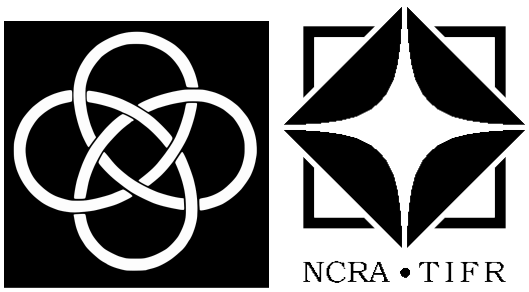
17th Radio Astronomy Winter School (RAWS) 2024



Submitted by (`Group - 3`)

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Experiment: Receiver Characteristics

1 Aim

In this experiment we aim

- To study the dynamic range of a transmission line by analyzing the interference between forward and reflected signals.
- To find the time delay between the interfering signals to calculate the length of the wire.

2 Experimental Setup

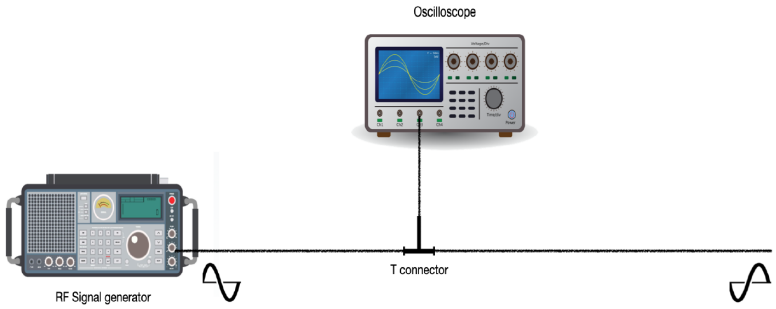


Figure 2: The setup of our experiment (Source: Manual).

- **RF Signal Generator:** They generate signals of desired frequency.
- **T-Connector:** It connects the signal generator, oscilloscope and the open-ended cable.
- **Oscilloscope:** Measures and displays the signal amplitudes at various frequencies.

3 Theory

The transmission lines are modeled as a series of inductors and capacitors per unit length, with a characteristic impedance, for analysis. Proper signal transport requires matching of the transmission line's characteristic impedance at either ends. When impedance mismatches occur at the ends, part of the signal reflects back, leading to interference between the forward (V_1) and reflected (V_2) waves. For accurate working of an electrical system, the instrument should be operated in its dynamic range. The output signal will be linearly proportional to the input ensuring no nonlinear cross-terms distort the signal. This requires working in a regime where the signal exceeds noise but remains below saturation, with the instrument set to nominal gain: dynamic range.

Consider the original wave V_1:

$$V_1 = V \sin(\omega t + \phi)$$

The reflected wave travels an additional distance, introducing a time delay (τ) and a phase shift (π) as given:

$$V_2 = \alpha V \sin\left[\omega(t+\tau) + \phi + \pi\right]$$

Here, α represents the total loss factor, or the amount of the wave reflected back.

Constructive or destructive interference happens on the T-connector depending on their relative phase when:

$$\omega t + \phi = \omega(t+\tau) + \phi + \pi \pm n2\pi$$

Which simplifies to

$$\omega\tau = \pi \pm n2\pi$$

If successive minima occur at f_1 and f_2,

$$\Delta f = \frac{1}{\tau}$$

Now, we can find the length of the cable using,

$$\tau = \frac{2x}{v_{\text{cable}}} \quad \Rightarrow \quad x = \frac{\tau v_{\text{cable}}}{2}$$

Next, we need to understand how much of the wave is reflected back, i.e to find (alpha). The maximum and minimum voltages would be

$$V_{\text{max}} = V + \alpha V, \quad V_{\text{min}} = V - \alpha V$$

Therefore (alpha) can be found as:

$$\alpha = \frac{(V + \alpha V) - (V - \alpha V)}{(V + \alpha V) + (V - \alpha V)}$$

The total loss factor per unit length is:

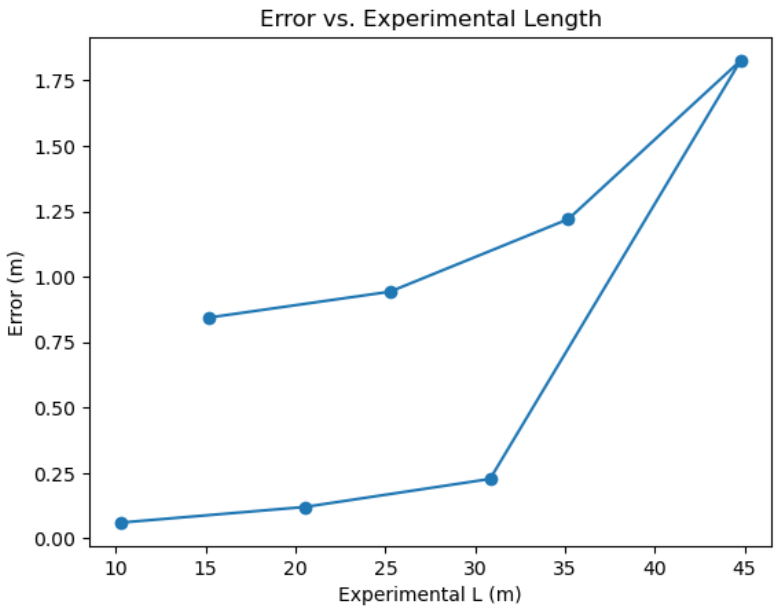
$$\alpha_{\text{per unit length}} = \frac{\alpha}{2x}$$

This experiment illustrates how even tiny impedance mismatches can cause reflections and standing waves, distorting signals. In radio astronomy, such mismatches severely impact the detection of weak cosmic signals, affecting spectral analysis.

4 Data Analysis

Observations on Length Measurements

Table in the appendix shows the values of lengths that were measured during the experiment (denoted by theoretical length) and the lengths that are calculated using the frequency difference (experimental length). The error is calculated as the difference between the two. From the measured values, it can be inferred that the error is increasing with increase in the experimental length. We can assume that the uncertainty in data is increasing due to systematic errors in the measurement process which became more pronounced as the length increased. The graph of error v/s experimental length is as shown:



It is evident that the maximum error is 1.825906 m at 46.6 m length.
The Empirical relation for the difference in distance traveled by the interfering waves and frequency can be demonstrated by the following graph:

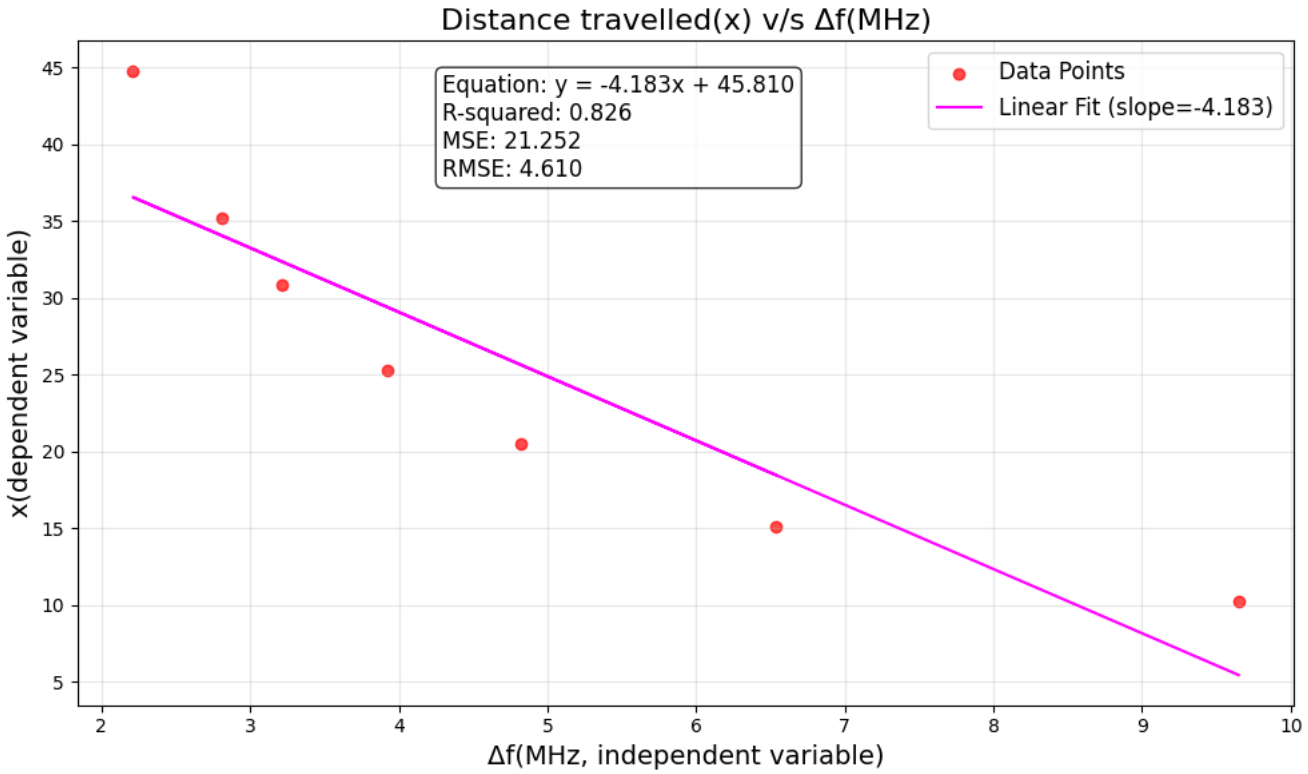


Figure 3: Empirical relation of x v/s Δf

- R-squared, also called the coefficient of determination, indicates how well the linear regression line fits your data.
- About **82.6%** of the variability in the dependent variable Δf is explained by the independent variable (x) using the linear regression model. This suggests a **strong linear relationship** between the two variables.
- MSE measures the average squared differences between the observed values and the predicted values from the linear regression model. A smaller MSE indicates that the regression line closely fits the data points. The value of 21.252 showcases that our model approximately fits the data points.
- RMSE is simply the square root of MSE, bringing the units back to the original scale of the dependent variable.

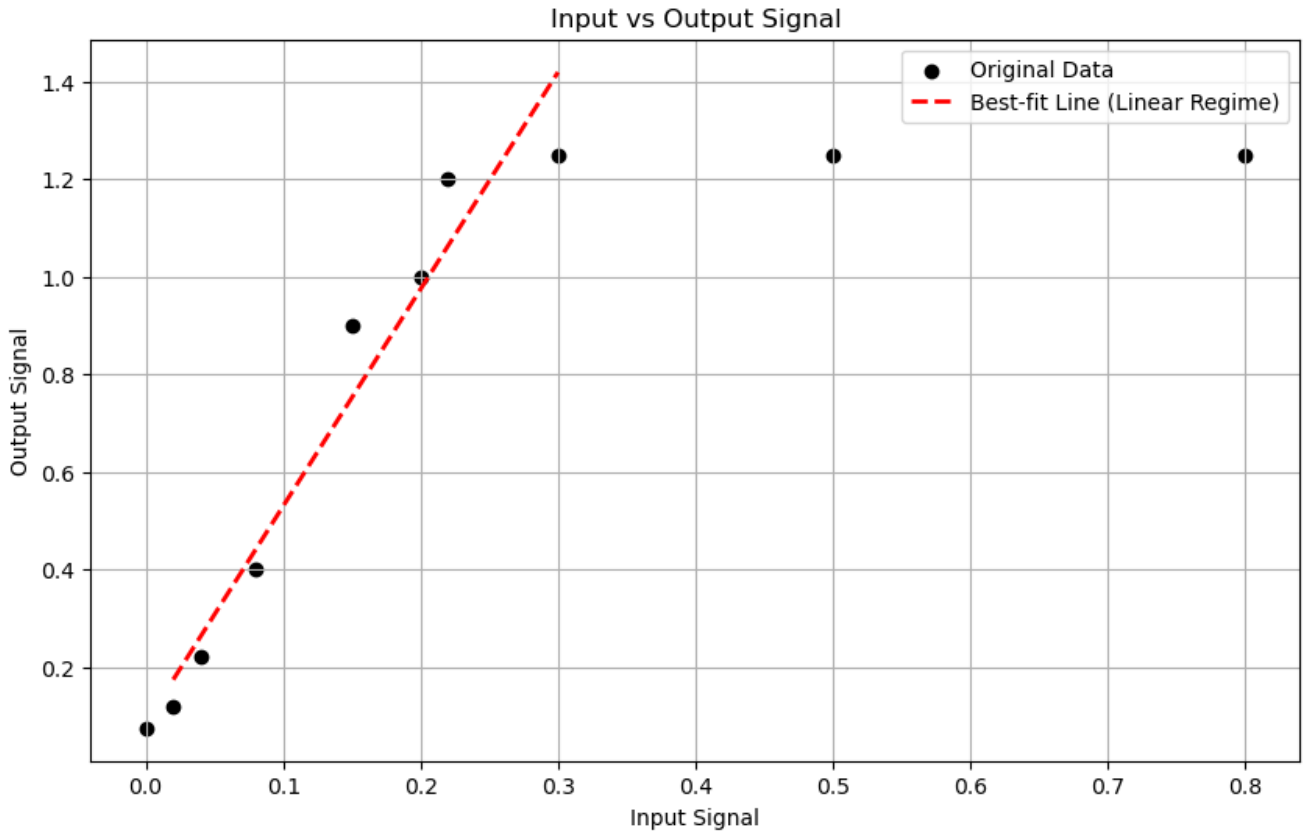
Output Signal v/s Input Signal

In the second part of the experiment, we measured the output voltage for different input voltages by varying the amplitude in nominal gain. Observations for various gains were tested. Then we plotted the graph with input voltage in the x-axis and output voltage in the y-axis to study the characteristics. From the graph, we can find the maximum and minimum voltages and hence the alpha value which is the total loss factor.

$$V_{\text{max}}\text{: } 1.25$$
$$V_{\text{min}}\text{: } 0.22$$
$$\alpha \text{ (total loss factor)} \approx 0.7$$

A high loss factor can negatively impact the system's performance. For example, in an amplifier, a high loss factor might result in a limited dynamic range and reduced output power.

The graph for gain=0.5 is given below.



The graph exhibits a non-linear relationship between the input and output voltages. Therefore, the gain is not constant across the input-range. We can see that the graph is not linear as the input voltage is increased beyond a certain level. The output voltage reaches a point where it saturates and remains relatively constant. This point decides the upper bound of the dynamic range of the amplifier. Now, we perform a linear fit to fit the data, excluding the points giving non-linear relationship.

The red dashed line represents the linear fit. The equation of the line is y= 4.45x + 0.09 where the slope of the graph is 4.45 and the y-intercept is 0.09. The y-intercept represents the value of the output voltage even when the input is zero. The linear fit suggests that, within the range of the input voltages considered, the output voltage can be reasonably approximated by a linear function of the input voltage.

To find how well the linear model fits the data, we calculated the Mean Squared Error which is the average squared difference between the actual values and the values predicted by the model.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(V_{\text{output},i} - (\text{slope} \cdot V_{\text{input},i} + \text{intercept})\right)^2$$

The calculated mean squared error is 0.0119. This suggests that within the dynamic range of the amplifier, the output voltage is linearly proportional to the input voltage at a particular constant gain. This is verified by the fact that the fit showed a linear relationship when we excluded the low-output offset and the saturation region and the mean squared error is less indicating the linear fit is a good approximation of the data.

5 Sources of Errors

- Impedance Mismatch: Inadequate impedance matching at the cable ends can cause partial signal reflections, causing inaccuracies in calculating the cable length.
- Measurement Errors: Errors in the manual recording of frequencies, voltages, or amplitudes may lead to errors in calculated results.

- Using an audio amplifier, designed for lower-frequency signals, instead of an RF amplifier introduced limitations. Audio amplifiers lack the bandwidth and stability required for precise high-frequency applications, likely contributing to measurement inaccuracies.

6 Conclusion

The experiment successfully demonstrated the relationships between theoretical and experimental lengths, error trends, and the input-output voltage behavior in the system. Key findings include:

- Length Measurements: Experimental lengths derived from frequency differences were in reasonable agreement with theoretical values, though systematic errors increased with length. The maximum observed error (1.825906m) occurred at the longest theoretical length (46.6m).
- Empirical Relationships: A strong linear correlation ($R^2 = 0.826$) was observed between Δf and the independent variable x, indicating the model captures most of the variability. The Mean Squared Error(21.252) and Root Mean Squared Error affirm the model's adequacy within the experimental data range.
- Input-Output Voltage Characteristics: The system exhibited non-linear behavior at higher input voltages, leading to saturation and defining the dynamic range of the amplifier. A linear fit ($y = 4.45x+0.09$) accurately described the relationship within the linear region, with a low MSE (0.0119).
- Loss Factor (α): The total loss factor (0.70068) reflects significant wave attenuation, which may affect the system's efficiency and performance in real-world applications.

7 Discussion

- Using an audio amplifier, designed for lower-frequency signals, instead of an RF amplifier introduced limitations. Audio amplifiers lack the bandwidth and stability required for precise high-frequency applications. This increased systematic errors, particularly for longer lengths. Employing an RF amplifier, designed for higher-frequency applications, and improving experimental conditions (e.g., better isolation from external noise) would reduce these errors.
- The saturation of the audio amplifier restricted the dynamic range and led to non-linear behavior at higher input voltages.
- Despite its limitations, the experiment provided valuable insights into error trends, amplifier characteristics, and the importance of equipment selection. These findings can inform future experiments in wave analysis and signal processing.
- Future experiments should utilize RF amplifiers to minimize equipment-related errors and extend the range of applicability. Additionally, automating measurements and incorporating advanced error correction techniques could further enhance accuracy. This revised conclusion and discussion account for the use of the audio amplifier as a source of error.

Appendix

S.No	l_{th} (m)	f_1 (MHz)	f_2 (MHz)	Velocity Factor	l_{exp} (m)	Error (m)
0	16.0	16.1318	22.6638	0.66	15.156154	0.843846
1	26.2	17.4375	21.3572	0.66	25.257035	0.942965
2	36.4	18.0406	20.8547	0.66	35.179986	1.220014
3	46.6	14.0204	16.2315	0.66	44.774094	1.825906
4	30.6	14.6281	17.8395	0.66	30.827676	0.227676
5	20.4	17.0353	21.8597	0.66	20.520687	0.120687
6	10.2	14.6234	24.2718	0.66	10.260769	0.060769

Table 1: Measurements for first part(length of the cable)

V_{in}	V_{out}
0	75 mV
20 mV	120 mV
40 mV	220 mV
80 mV	400 mV
150 mV	0.9 V
2 V	10 V
2.12 V	12 V
3 V	12.5 V
5 V	12.5 V
8 V	12.5 V

Table 2: Measurements for second part(Saturation levels)