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Subj: PH3203 | ASSGN 03.

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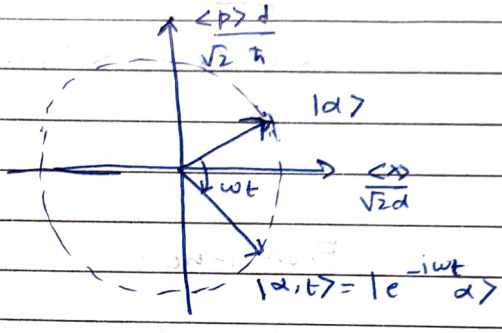
Explicitly calculate.

$$\langle\alpha|\,\hat{\phi}\,|\alpha\rangle = \langle\phi\rangle$$

and.

$$\Delta\hat{\phi} = \left[\langle\alpha|\,\hat{\phi}^2\,|\alpha\rangle - (\langle\alpha|\,\hat{\phi}\,|\alpha\rangle)^2\right]^{1/2}.$$

Show $\Delta\hat{\phi} \approx \frac{1}{\Delta N}$.



Ans Define

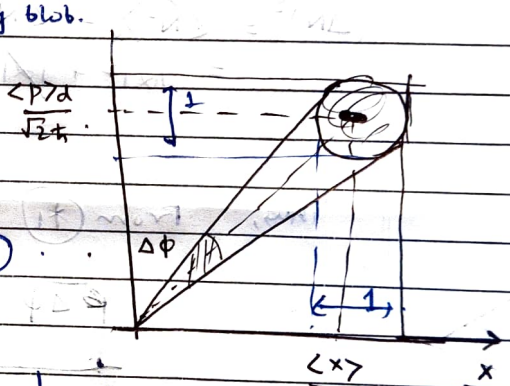
$$d \equiv \sqrt{\frac{\hbar}{m\omega}}$$

$\Delta\phi$:= phase ambiguity of blob.

Clearly from diagram.

$\Delta\phi \cdot |\alpha| \simeq 1$.

$$\Rightarrow \Delta\phi \simeq \frac{1}{|\alpha|} \quad \text{---} (*)$$



Claim 1: Diameter of blob = 1.

We know, position & mom. uncertainity for coherent state is same as for the ground state.

$$\Delta x = \frac{d}{\sqrt{2}}\,;\quad \Delta p = \frac{\hbar}{d\sqrt{2}}.$$

When doing measurement on $|\alpha\rangle$, the uncertainities on value of α can be represented by a blob of unit diameter centered at α.

∴ For state $|\alpha\rangle$.

Repr. range for measured $x \in \left[\langle x\rangle - \frac{1}{2}\Delta x, \langle x\rangle + \frac{1}{2}\Delta x\right]$.

width = 1 ⟸ Repr. " " " $\frac{x}{\sqrt{2}d} = \left[\frac{\langle x\rangle}{\sqrt{2}d} - \frac{1}{2}, \frac{\langle x\rangle}{\sqrt{2}d} + \frac{1}{2}\right]$

Similarly for momentum

$$p \in \left[\langle p\rangle - \frac{1}{2}\Delta p, \langle p\rangle + \frac{1}{2}\Delta p\right]$$

Width = 1 ⟸ Repr. range for measured $\frac{pd}{\sqrt{2}\hbar} = \left[\frac{\langle p\rangle d}{\sqrt{2}\hbar} - \frac{1}{2}, \frac{\langle p\rangle d}{\sqrt{2}\hbar} + \frac{1}{2}\right]$.

Hence, Proved. □

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Claim 2: $\Delta N = |\alpha|$ for state $|\alpha\rangle$

$$\langle \hat{N} \rangle_\alpha = \langle \alpha | \hat{N} | \alpha \rangle = \langle \alpha | a^\dagger a | \alpha \rangle = \langle \alpha | \alpha^* \alpha | \alpha \rangle = |\alpha|^2$$

~~To find~~

$$\langle \hat{N}^2 \rangle_\alpha = \langle \alpha | a^\dagger a a^\dagger a | \alpha \rangle = |\alpha|^2 \langle \alpha | a\, a^\dagger | \alpha \rangle$$

$$= |\alpha|^2 \left(\langle \alpha | ([a, a^\dagger] + a^\dagger a) | \alpha \rangle \right)$$

$$= |\alpha|^2 (1 + |\alpha|^2).$$

From, Phase.

$$(\Delta N)^2 = \langle \hat{N}^2 \rangle_\alpha - \langle \hat{N} \rangle_\alpha^2$$

$$= |\alpha|^2 + |\alpha|^4 - |\alpha|^4 \Rightarrow \boxed{\Delta N = |\alpha|}$$

Thus, From (*1)

$$\boxed{\Delta \phi \geq \frac{1}{\Delta N}}$$

This is a familiar relation for coherent states of light. It then relates the uncertainty in the number of photons to uncertainty in phase of the wave.

$\langle\alpha|\hat{\phi}|\alpha\rangle = ?$

We know.

$|\alpha\rangle = \hat{D}(\alpha)|0\rangle$

$\therefore \langle\hat{\phi}\rangle_\alpha = \langle 0|\hat{D}^\dagger(\alpha)\hat{\phi}\hat{D}(\alpha)|0\rangle$

where $\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$. → Displacement operator.

In order to find this, we look to evaluate.

$[\hat{D}(\alpha), e^{-i\hat{\phi}}]$

$= [\hat{D}(\alpha), \frac{\hat{a}}{\sqrt{\hat{N}+1}}]$

$= [\hat{D}(\alpha), \hat{a}]\frac{1}{\sqrt{\hat{N}+1}} + \hat{a}[D(\alpha), \frac{1}{\sqrt{\hat{N}+1}}]$

$= \frac{-\alpha\hat{D}(\alpha)}{\sqrt{\hat{N}+1}} + \hat{a}\left[\hat{D}(\alpha), \frac{1}{\sqrt{\hat{N}+1}}\right]$

$[\hat{D}(\alpha), \hat{N}] = e^{-|\alpha|^2/2}[e^{\alpha a^\dagger} \cdot e^{-\alpha^* a}, a^\dagger a]$

opening

$= e^{-|\alpha|^2/2}\{a^\dagger[e^{\alpha a^\dagger}, a]e^{-\alpha^* a} + e^{\alpha a^\dagger}[e^{-\alpha^* a}, a^\dagger]a\}$

Evaluating

$[e^{\alpha a^\dagger}, a] = -\alpha e^{\alpha a^\dagger}$

$[e^{-\alpha^* a}, a^\dagger] = -\alpha^* e^{-\alpha^* a}$.

$$\therefore [\hat{D}(\alpha), \hat{N}] = e^{-|\alpha|^2/2} \{ -\alpha a^\dagger e^{\alpha a^\dagger} \cdot e^{-\alpha^* a} + e^{\alpha a^\dagger} e^{-\alpha^* a} (-\alpha^* a) \}$$

$$= -(\alpha a^\dagger + \alpha^* a) \hat{D}(\alpha)$$

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