Diodes

Susnata Chattopadhyay[1], 22MS077 December 9, 2024

Problem 2.1 Find the bias for which the reverse current in a silicon (Si) $p-n$ junction diode is half its saturation value at room temperature.

Solution: We begin with the equation of continuity for holes in a semiconductor

$$\frac{\partial p}{\partial t} = \frac{p_0 - p}{\tau_p} - \frac{1}{q}\frac{\partial J_p}{\partial x}$$

This expression can be explained as follows: there is an *increase* per second of $g = p_0/\tau_p$ holes per unit volume due to thermal generation, and a *decrease* per second of p/τ_p holes per unit volume due to recombination. Furthermore, due to a nonuniform concentration $p(x)$ across the junction, we also have a diffusion hole current density $J_p = -qD_p\frac{\mathrm{d}p}{\mathrm{d}x}$ $[p \to n]$ (D_p is the diffusion constant). So for carrier charge q, we again have a *decrease* in hole concentration per second due to current I_p

$$\frac{\mathrm{d}I_p/q}{A\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}J_p}{\mathrm{d}x}$$

τ_p being the mean lifetime of a hole. From **Einstein Relationship** we define the *volt-equivalent of temperature*

$$V_T \equiv \frac{\bar{k_B}T}{q} = \frac{T}{11600}$$

where, $\bar{k_B}$ is Boltzmann constant in Joules per Kelvin. The **diffusion length** for holes L_p by

$$L_p \equiv \sqrt{D_p\tau_p}$$

Now, solving from the boundary conditions of the differential equation for injected hole concentration $p' = p - p_0$

$$\frac{\mathrm{d}^2p'}{\mathrm{d}x^2} = \frac{p'}{L_p^2}$$

we have

$$p'(x) = p'(0)e^{-x/L_p} = p(x) - p_0 \implies p(x) = p_0 + p'(0)e^{-x/L_p} \tag{1}$$

So, from eq 1, and using the definition of J_p, we obtain

$$I_p(x) = \frac{AqD_pp'(0)}{L_p}e^{-x/L_p} = \frac{AqD_p}{L_p}\left[p(0) - p_0\right]e^{-x/L_p} \tag{2}$$

Thus the minority (hole) diffusion current at the junction ($x = 0$) is given by

$$I_{pn}(0) = \frac{AqD_p}{L_p}\left[p_n(0) - p_{n0}\right]$$

Now, a forward bias V lowers the barrier height and allows more carriers to cross the junction. From the Boltzmann relationship (of kinetic gas theory), it seems reasonable that $p_n(0)$ should depend exponentially upon V.

$$p_n(0) = p_{n0}e^{V/V_T}$$

[1]Email: sc22ms077@iiserkol.ac.in

This is called the **law of junction**.
Substituting we obtain

$$I_{pn}(0) = \frac{AqD_p p_{n0}}{L_p}(e^{V/V_T} - 1)$$

Finally after experimental corrections, we write the **total diode current**

$$\boxed{I = I_0(e^{V/\eta V_T} - 1)}$$

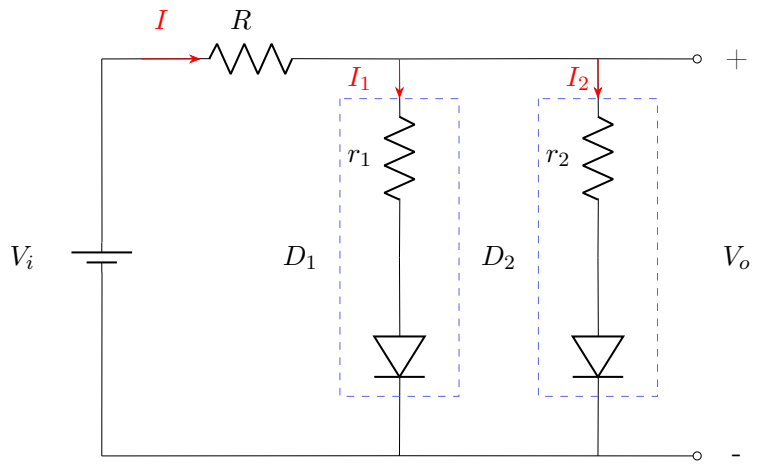
where I_0 is the magnitude of reverse saturation current. The constant η (called *diode ideality factor* or the *emission coefficient*) is unity for germanium and is approximately 2 for silicon at rated current. At room temperature (300 K) $V_T \approx 26$ mV.

In this problem, we are asked to find the bias V for which $I = -I_0/2$ for Si diode,

$$V = 2V_T \ln(1 + I/I_0) \implies = -2 \times 26 \ln 2 \simeq \boxed{-36.04 \text{ mV}}$$

Ans

Problem 2.2 Consider the following network with $R = 500\ \Omega$ and $V_i = 5$ V. The cutin voltages $(V_{\gamma 1}, V_{\gamma 2})$ and forward resistances (r_1, r_2) for diode D_1 and D_2 are (0.7 V, 0.6 V) and (50 Ω , 75 Ω) respectively. Calculate (i) the currents I_1, I_2 and I (ii) the output voltage V_o and (iii) the powers delivered to D_1 and D_2.



Solution: Since, the diodes have a forward resistance we explicitly showed them for the convenience of our calculations. We first try to understand **Cut-in voltage** of a particular diode.

Cut-in Voltage In practical Si and Ge diodes, it is found that the forward current does not assume a significant value unless the forward bias has exceeded a certain threshold value V_γ , called cut-in voltage[a]. Below V_γ , current is less than 1% of maximum rated value and beyond V_γ, current rises very rapidly. The existence of V_γ can be explained from the intrinsic potential barrier V_b existing across the junction. The current through the diode becomes appreciable only when the forward voltage exceeds V_b. Thus V_γ equals the forward voltage for which the effective barrier vanishes. Therefore,

$$V_\gamma = V_b$$

Please try to appreciate, that the value of V_γ depends on the doping concentrations as well as the temperatures .

[a] *Fundamental Principles of Electronics*, B. Ghosh, pg- 103

Applying the usual KVL and current conservation we have

$$V_i - RI - r_1 I_1 - V_{\gamma 1} = 0 \implies 4.3 = 500I + 50I_1 \tag{3}$$
$$V_i - RI - r_2 I_2 - V_{\gamma 2} = 0 \implies 4.4 = 500I + 75I_2 \tag{4}$$
$$V_{\gamma 1} + r_1 I_1 - r_2 I_2 - V_{\gamma 2} = 0 \implies -0.1 = 50I_1 - 75I_2 \tag{5}$$
$$I - I_1 + I_2 = 0 \tag{6}$$

(i) Eliminating I, and solving the above system of equations give $\boxed{I_2 = 4.0755 \text{ mA}}$, $\boxed{I_1 = 4.1132 \text{ mA}}$ and consequently $\boxed{I = 4.0755 + 4.1132 = 8.1887 \text{ mA}}$

(ii) The output voltage will be $V_o = V_i - RI = 5 - 0.5 \times 8.1887 \simeq 0.906$ volts **Ans**

(iii) The power delivered to a diode (with forward resistance) in a DC current is given by,

$$P = V_\gamma I_D + I_D^2 R_f$$

Hence, $P_1 = 0.7I_1 + 50I_1^2 \implies \boxed{P_1 = 3.7252 \text{ mW}}$ and $P_2 = 0.6I_2 + 75I_2^2 \implies \boxed{P_2 = 3.6910 \text{ mW}}$

Thus overall current handling capacity is increased by such parallel connection. It also increases the overall wattage rating.

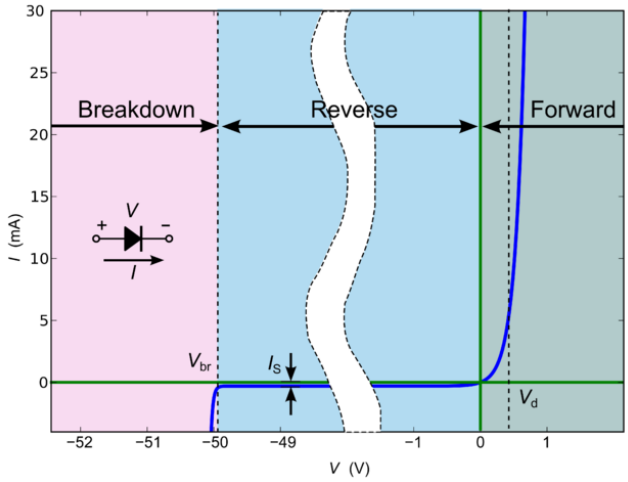


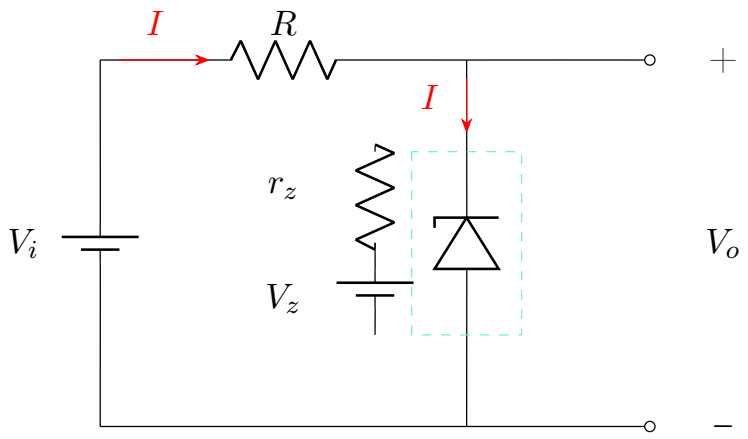
Figure 1: Different operating regions of a diode. Source: Wikipedia

Problem 2.3 Considering the following circuit with $R = 800\ \Omega$, the Zener diode is non-ideal, having a knee voltage $V_z = 9$ V and a dynamic resistance $r_z = 5\Omega$. If the input voltage varies from 15 V to 30 V, determine the range of variation of the output voltage V_o. Comment on the result.

Solution: Diodes which are designed with adequate power-dissipation capabilities to operate in the *breakdown region* may be employed as voltage-reference or constant voltage devices. Such diodes are known as *avalanche, breakdown* or *zener* diodes[2]. Since, we are given the zener diode to be non-ideal, we replace it by it's equivalent knee voltage and dynamic resistance combination. We begin by applying KVL on the two loops:

$$V_i - 800I - V_o = 0 \tag{7}$$
$$V_i - 800I - r_z I - V_z = 0 \implies I = \frac{V_i - 9}{805} \tag{8}$$

[2] *Integrated Electronics* by Millman and Halkias, Pg-73



Substituting I in the above equation gives,

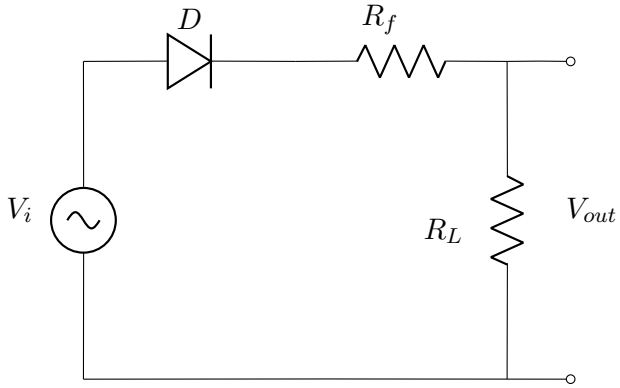
$$V_o = V_i - \frac{160}{161}(V_i - 9) = \frac{V_i}{161} + 8.944$$

Thus, on varying input voltage V_i from 15 V to 30 V, we have $\Delta V_i = 15$ V. So,

$$\Delta V_o = \frac{\Delta V_i}{161} = \frac{15}{161} = 0.093167 \simeq \boxed{0.093 \text{ V}}$$

We find that, the presence of diode stabilises the circuit. Because we see a large variation in input voltage ($\Delta V_i = 15$ V), produces a very small fluctuation ($\Delta V_o = 0.093$ V) in the output voltage.

Problem 2.4 A diode having forward resistance of 50 Ω , supplies power to a load resistance 1200 Ω from a 20 V (rms) source. Calculate (in) the DC load current, (i) the rms value to AC load current, (iii) the DC voltage across the diode, (iv) the dc output power, (v) the conversion efficiency, and the percentage regulation.



Solution:

(i) We are given $V_{rms} = V_0/\sqrt{2} = 20$ V. Let the instantaneous input voltage be

$$V_i = V_0 \sin \omega t$$

The peak value of voltage V_0 is usually large as compared to the cut-in voltage V_γ of the diode. Thus for simplicity we may assume $V_\gamma = 0$. Hence, the current through the diode and load R_L will be given by

$$\begin{aligned} i &= I_0 \sin \omega t && 0 \le \omega t \le \pi \\ &= 0 && \pi \le \omega t \le 2\pi \end{aligned}$$

where $I_0 = \frac{V_0}{R_f + R_L}$ is the peak value of load current. The d.c. value of load current is defined as

$$I_{dc} = \frac{1}{2\pi} \int_0^{2\pi} i \, \mathrm{d}(\omega t)$$

Substituting i from the above equation and integrating in correct limits gives us

$$\boxed{I_{dc} = \frac{I_0}{\pi}}$$

Calculating gives us $I_{dc} = \frac{22.63}{\pi} \simeq 7.20$ mA. **Ans**

(ii) The r.m.s value to the AC load current is given by

$$\begin{aligned} I_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 \, \mathrm{d}(\omega t)} \\ &= \sqrt{\frac{I_0^2}{2} \times \frac{1}{\pi} \int_0^{\pi} \sin^2 \omega t \, \mathrm{d}(\omega t)} \\ &= \sqrt{\frac{I_0^2}{2} \times \frac{1}{2}} = \frac{I_0}{2} \end{aligned}$$

Calculating, $I_{rms} = 22.63/2 = 11.31$ mA. **Ans**

(iii) The reading of a d.c voltmeter across a diode is not just simply $I_{dc}R_f$, because the diode cannot be modeled as a constant resistance, but rather it has two values: R_f in the *ON* state and ∞ in the *OFF* state.

Note that a d.c voltmeter reads the average value of voltage across the terminals of the diode. Hence, to obtain V'_{dc} across the diode, the instantaneous voltage must be plotted as in Fig. 2 and the average value obtained by integration.

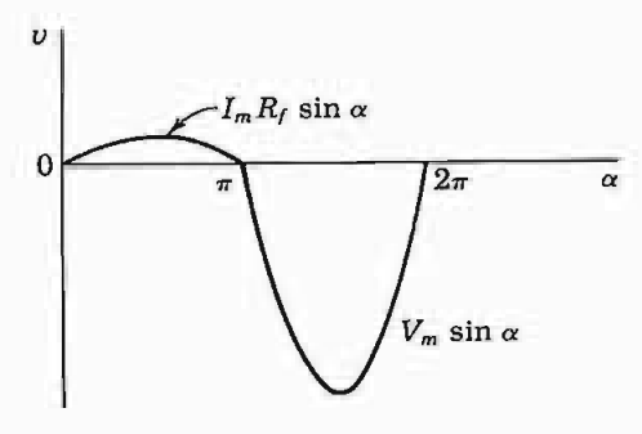


Figure 2: The voltage across the diode ($\alpha = \omega t$; $I_m = I_0, V_m = V_0$)

Thus,

$$\begin{aligned} V'_{dc} &= \frac{1}{2\pi} \left(\int_0^{\pi} I_0 R_f \sin\alpha \, \mathrm{d}\alpha + \int_{\pi}^{2\pi} V_0 \sin\alpha \, \mathrm{d}\alpha \right) \\ &= \frac{1}{\pi}(I_0 R_f - V_0) = \frac{1}{\pi}[I_0 R_f - I_0(R_f + R_L)] \\ &= -I_0 R_L/\pi \end{aligned}$$

$$\therefore \; V'_{dc} = -I_{dc} R_L = -7.20 \times 1200 \times 10^{-3} = -8.64\text{V}$$

The $'-'$ sign indicates a voltage drop.

(iv) The total DC output power is the power supplied to the load

$$P_{dc} = I_{dc}^2 R_L = 62.208 \text{ mW}$$

(v) The *conversion efficiency* or the *efficiency of rectification* (η) measures the total input power which is converted into d.c

$$\begin{aligned}\eta &= \frac{P_{dc}}{P_{in}} \times 100\% \\ &= \frac{I_{dc}^2 R_L}{I_{rms}^2 (R_L + R_f)} \times 100\% \\ &= \frac{40.6}{1 + \frac{R_f}{R_L}}\%\end{aligned}$$

Calculating we have, $\eta \simeq 38.97\%$ **Ans**

The variation of d.c output voltage with d.c load current for a given input is called the load regulation of the rectifier. The degree of stabilization of the output is usually expressed in terms of **percentage regulation**(S_L).

$$S_L = \frac{V_{NL} - V_L}{V_L} \times 100\%$$

V_{NL} is the no-load voltage (i.e with $I_{dc} = 0$), while $V_L = V_{dc}$. We would like to deduce a relationship between V_{dc} and I_{dc}

$$I_{dc} = \frac{V_0}{\pi(R_f + R_L)} \implies I_{dc}R_L \equiv V_{dc} = \frac{V_0}{\pi} - I_{dc}R_f$$

Thus,

$$V_{NL} = \frac{V_0}{\pi} \; ; \; V_L = \frac{V_0}{\pi} - I_{dc}R_f$$

Substituting in the above expression gives us,

$$\boxed{S_L = \frac{R_f}{R_L} \times 100\%}$$

Calculating, $S_L = 4.17\%$. **Ans**

Problem 2.5 The output voltage across a load resistance $R_L = 100\ \Omega$ of a capacitor filter with C = 1100 μF connected to a full-wave rectifier supply with a line frequency of 50 Hz has a dc value of 9 V and a peak-to-peak ripple voltage of 0.8 V. Calculate the ripple factor from the voltage values and compare it with the theoretical value expected from the filter components.

Solution: We are given $V_{dc} = 9$ V, $V_{p-p} = 0.8$ V, $f = 50$ Hz, $R_L = 100\ \Omega$, C = 1100 μF .

(i) **Theoretical Value**

Since $\omega C R_L = 11\pi$ which is much larger than 1. The theoretical ripple factor can be given by

$$\gamma_{th} = \frac{1}{4\sqrt{3} f C R_L} = \frac{1}{22\sqrt{3}} \simeq 0.0262$$

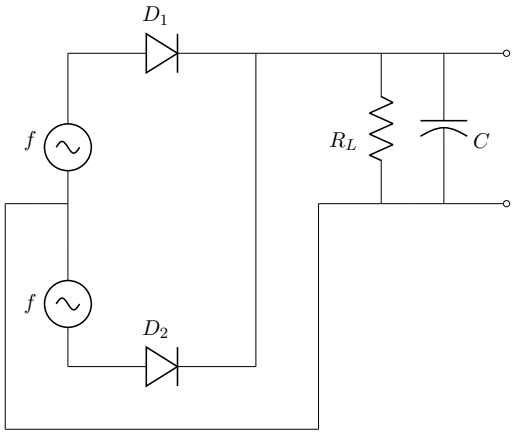


Figure 3: Equivalent circuit of a full wave rectifier with capacitor filter

(ii) **Observed Value**

The r.m.s value of the assumed triangular ripple voltage, and the d.c value of the triangular waveform respectively is approximately given by

$$V'_{r.m.s.} = \frac{V_{p-p}}{2\sqrt{3}}$$

$$V_{dc} = V_0 - \frac{V_{p-p}}{2}$$

Therefore, the ripple factor

$$\gamma_V = \frac{V'_{r.m.s.}}{V_{dc}} = \frac{0.8}{2\sqrt{3} \times 9} \simeq 0.0257$$

Hence, we find the observed ripple factor from voltage components to be slightly less than the theoretical value.

Ripple Factor (γ) In a full wave rectifier the output current waveform can be written as

$$i = I_0|\sin \omega t| \qquad 0 \le \omega t \le 2\pi$$

The purpose of the rectifier is to produce a smooth d.c but here the output is pulsating in nature. In fact, by making a Fourier analysis of the output waveform we can show that the output contains d.c as well as harmonic components -

$$i = I_0\left(\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t - \dots\right)$$

A measure of these fluctuating components, and hence the waviness or lack of smoothness of the output is expressed in terms of ripple factor γ which is defined as

$$\gamma = \frac{\text{r.m.s. value of a.c. components of load current (or voltage)}}{\text{d.c. value of load current (or voltage)}}$$

To find γ, write

$$i = I_{dc} + i_{ac} \quad \text{or} \quad i_{ac} = i - I_{dc}$$

Now we have to find $I'_{r.m.s.}$ of the output ac component i_{ac}.

$$I'_{r.m.s.} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (i - I_{dc})^2 \, \mathrm{d}(\omega t)}$$

Finally calculating we have

$$\gamma \equiv \frac{I'_{r.m.s.}}{I_{dc}} = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \frac{V'_{r.m.s.}}{V_{dc}}$$

Problem 2.6 Consider the following Clipping circuit with R = 4kΩ , V_R = 3 V and the diode is an ideal one. The input voltage V_i is a triangular waveform as given below. Find the output voltage (V_{out}) and the diode current (i_D) waveforms.

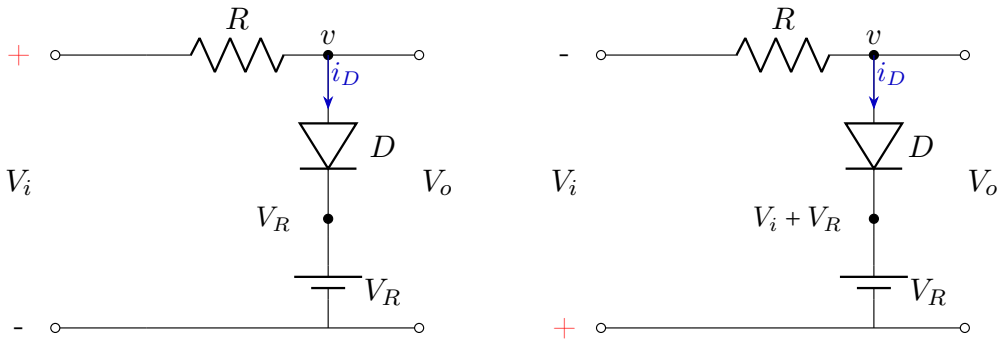


Figure 4: The cliipping circuit in **Left:** positive half cycle and, **Right:** negative half cycle.

Solution: We analyze the circuit in the two cycles as depicted above.

Case-1 +ve cycle

Let V_i increase from 0. And the voltage at the junction node is v.

(i) $v < V_R \implies$ Notice diode is reversed biased, so $i_D = 0$. And thus $\boxed{V_o = V_i}$.

(ii) $v \geq V_R \implies$ As soon as v starts to increase beyond V_R, the diode becomes forward biased and starts to conduct. And since, the diode is ideal we have $v = V_R$, and hence $\boxed{V_o = V_R}$. The current in the diode

$$i_D = \frac{V_i - V_R}{R} = \frac{V_i - 3}{4} \text{ mA}$$

Case-2 -ve cycle

Notice, here $v = 0$ and $V_i + V_R > 0$, so the diode always remains reversed bias and

$$i_D = 0 \ ; \ V_o = V_i$$

We draw the diagram in the next page.

Clippers Clipping circuits are used to 'clip-off' a portion of the input signal. It is also called *voltage/ current limiter*, *amplitude selector* or *slicer*. It is useful for signal shaping (chopping off noise pulses in radio receivers), circuit protection and communications. Modified double diode clipper may also be used for the protection of a meter movement against over voltages.

Clampers However, the clamping circuits simply 'clamp' the input signal to a different dc level. They are used in TV receivers as d.c restorers of video signals. Clampers are also used in radar and communication circuits.

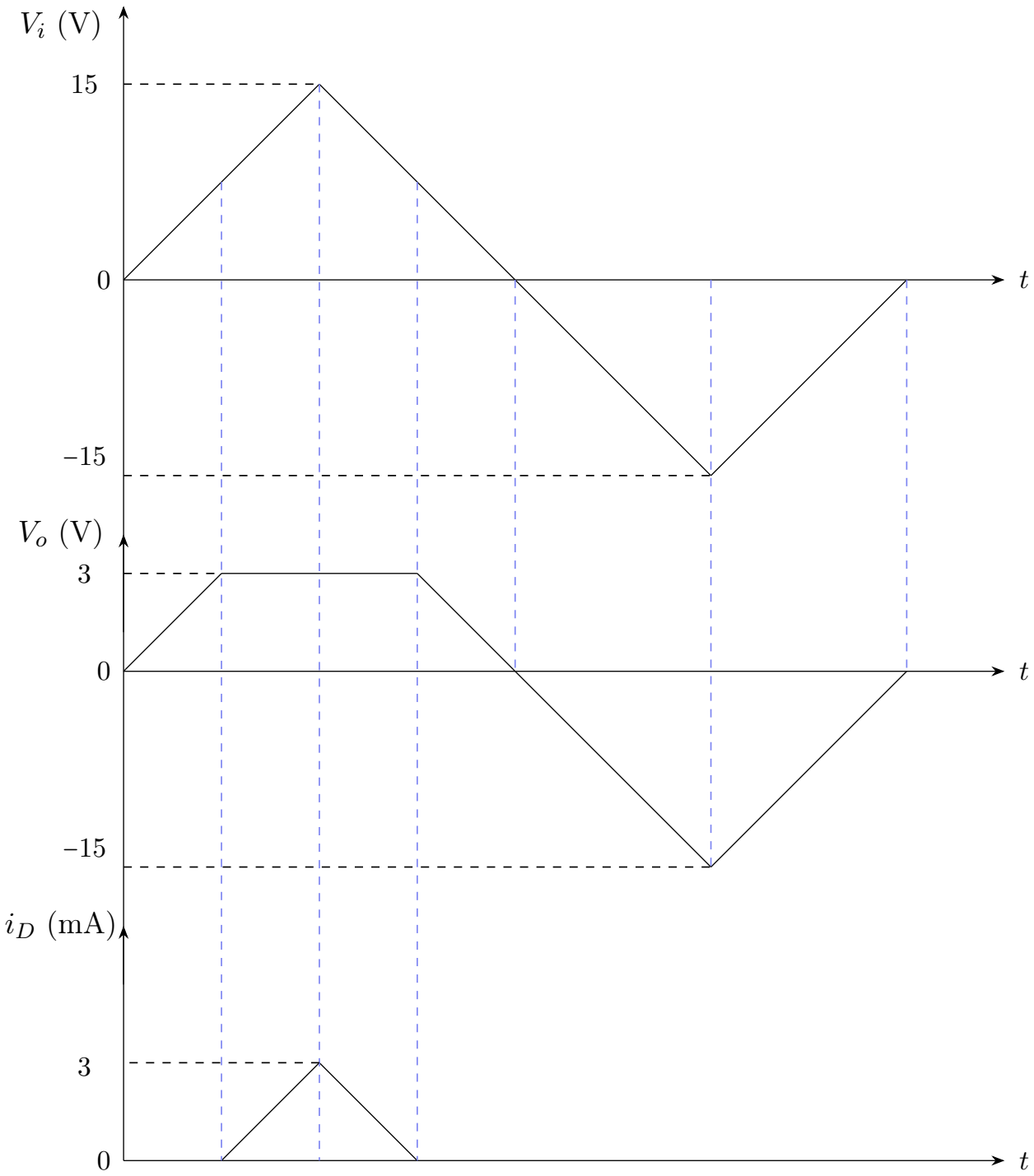


Figure 5: Top: Input voltage waveform, **Middle:** Output voltage waveform, **Bottom:** Diode current waveform