PH2202

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2) For a Van-der-Waal's gas equation of state is given by

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT \quad \text{---} (*)$$

Consider $U = \psi(T, V)$

$$\therefore dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

From 1st Law

$$dU = dQ - PdV \Rightarrow dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(P + \left(\frac{\partial U}{\partial V}\right)_T\right) dV$$

$$\Rightarrow dQ = nC_V dT + \left(P + \frac{an^2}{V^2}\right) dV = 0 \text{ (Adiabatic)}$$

Claim: $\left(\frac{\partial U}{\partial V}\right)_T = \frac{an^2}{V^2}$

We have, $dU = TdS - PdV$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$$

From Maxwell's Relation,

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P = \frac{an^2}{V^2}$$

From (*),

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$T\left(\frac{\partial P}{\partial T}\right)_V = \frac{nRT}{V - nb} + 0$$

D.

This follows,

$$-n C_V dT = \frac{nRT}{V - nb} dV$$

$$n = 1 \therefore -\frac{C_V}{R} \int \frac{dT}{T} = \int \frac{dV}{V - b} \Rightarrow -\frac{C_V}{R} \ln T = \ln(V - b) + K$$

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Further ~~$C_p - C_v = R$~~ . ~~$\frac{C_p}{C_v} = \gamma$~~ .

$$\therefore \frac{-1}{\gamma - 1} \int \frac{dT}{T} = \int \frac{dV}{V - nb} \Rightarrow T \cdot (V-b)^{C_v/R} = K = f(T,V)$$

$$\Rightarrow \frac{-1}{\gamma - 1} \ln T = \ln (V - nb) + K$$

$$\Rightarrow \left(T^{\frac{1}{\gamma - 1}}\right)(V - nb) = K \equiv f(T, V)$$

In adiabatic process,

$n = 1$,

$dQ = 0 = dU + \delta W$

$\therefore \delta W = -dU$

$\Rightarrow W = -\Delta U$

$$= -\left[C_v \Delta T + \int_{V_1}^{V_2} \frac{a}{V^2} \cdot dV \right]$$

$$= -\left[-C_v \Delta T + a \left[\frac{1}{V_2} - \frac{1}{V_1} \right] \right]$$

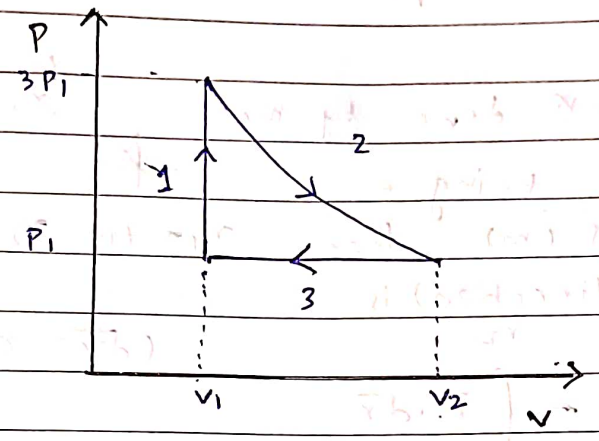
$$= \left\{ C_v (T_1 - T_2) + a \left[\frac{1}{V_2} - \frac{1}{V_1} \right] \right\}$$

Work done,

$$W = \frac{C_v}{R} \left[\left(P_1 + \frac{a}{V_1^2}\right)(V_1 - b) - \left(P_2 + \frac{a}{V_2^2}\right)(V_2 - b) \right] + a \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

(3) moles of O_2 = $\frac{3L \times 10^3}{3L} = 10^3$

$R = 8.314 \frac{J}{mol-K}$, $P_1 = 5$ atm.



$3P_1V_1 = P_1V_2$

Here,

Oxygen is a diatomic ideal Gas.

$C_v = \frac{5R}{2}$

$P_1V_1 = nRT_1$

$5 \times 10^5 V_1 = 10^3 \times 8.314 \times 300$.

$V_1 \approx 5\ m^3$.

In step 1

$W = 0$ (As Isochoric Process)

$\Delta U = n C_v \Delta T$

$= 10^3 \times \frac{5R}{2} \left(\frac{P_2V_2}{nR} - \frac{P_1V_1}{nR} \right)$

$= \frac{5}{2} \times 2 \times 5 \times 10^5 \times 5 = 12500$ kJ.

In step 2

$\Delta U = 0$ ($\Delta T = 0$)

$W = +3P_1V_1 \ln \frac{V_2}{V_1} = 3 \times 10^3 \times 8.314 \times 300 \ln \left(\frac{3V_1}{V_1} \right)$

$= 8220.47$ KJ

In step 3

$W = +P_1(V_1 - V_2) = 5 \times 10^5 (-2V_1)$

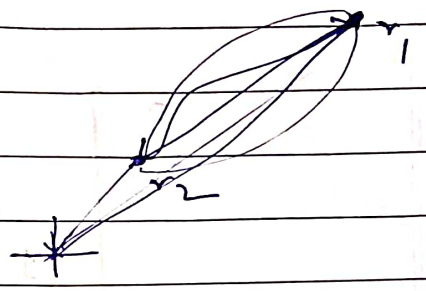
$= -10 \times 10^2 \times 5 = -5000$ kJ

$\Delta U = 10^3 \times \frac{5R}{2} \left(\frac{P_1V_1}{nR} - \frac{P_1V_2}{nR} \right)$

$= \frac{5}{2} \times 5 \times 10^5 (-2 \times 5) = -12500$ kJ

Q1) Gravitational field due to a mass 'M', (attracting) towards itself is given by

$$\bar{g} = -\frac{GM}{r^2}\hat{r}$$



Now work done by the field to bring a test mass (m) from r_1 to r_2 (in radial direction) is

$$W = -\int_{r_1}^{r_2} \bar{F} \cdot d\bar{r} \qquad (d\bar{r} = dr\,\hat{r})$$

$$= -\int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = \left.\frac{GMm}{r}\right]_{r_1}^{r_2}$$

$$= GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Thus the final work done depends only one the endpoints, and not on the practical path.

Thus work done in gravitational field is exact differential. (dW).