ASSIGNMENT-6

PH2201

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~~Class~~ Roll- 22MS077.

GROUP-2

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Q1) show that in momentum representation,

$$\hat{p}\,\psi(p) = p\,\psi(p)$$

the position $\hat{x}$ can be expressed as

$$\hat{x}\,\psi(p) = -\frac{\hbar}{i}\frac{d}{dp}\psi(p) - g(p)\,\psi(p)$$

where $g(p)$ is an ~~the~~ Arbitrary function.

A The canonical Commutation Relation states that $[\hat{x}, \hat{p}] = i\hbar$

To verify whether the given $\hat{x}$ satisfies this relation,

i.e $[\hat{x}, \hat{p}]\psi(p) \overset{?}{=} i\hbar\,\psi(p)$

LHS

$$[\hat{x}, \hat{p}]\psi(p) = \hat{x}\hat{p}\psi(p) - \hat{p}\hat{x}\psi(p)$$

$$= \hat{x}(p\,\psi(p)) - \hat{p}(\hat{x}\,\psi(p))$$

$$= \left(-\frac{\hbar}{i}\frac{d}{dp} - g(p)\right)(p\,\psi(p)) - \hat{p}\left[-\frac{\hbar}{i}\frac{d\psi(p)}{dp} - g(p)\,\psi(p)\right]$$

$$= \left[-\frac{\hbar}{i}\left[\psi(p) + p\frac{d\psi(p)}{dp}\right] - p\,g(p)\,\psi(p)\right] - \left[-\frac{\hbar}{i}p\frac{d\psi(p)}{dp} - g(p)\,p\,\psi(p)\right]$$

$$= i\hbar\,\psi(p) - \frac{\hbar}{i}p\frac{d\psi(p)}{dp} - p\,g(p)\,\psi(p) + \frac{\hbar}{i}p\frac{d\psi(p)}{dp} + g(p)\,p\,\psi(p)$$

$$= i\hbar\,\psi(p) = \text{RHS}$$

Hence, this form of $\hat{x}$ is a valid representation

□