H/W Assignment (3)

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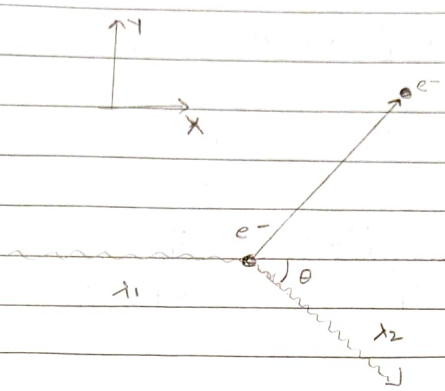
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Q) In the below example of Compton Scattering, show

$$\lambda_2 - \lambda_1 = \frac{h}{m_e}(1-\cos\theta)$$

(Take $c=1$)

Note that: The quantity $\left(\frac{h}{m_e c}\right)$ is known as compton wavelength.



Ans We have 4vector Momentum.

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right)$$

Note that

$$\|p^\mu\|^2 = p^\mu \cdot p_\mu = \left(\frac{E}{c}\right)^2 - |\vec{p}|^2 = m_0^2 c^2$$

= Lorentz Invariant.

where m_0 = rest mass of particle.

Let Before collision,

photon → p_1^μ

e^- → p_2^μ

After collision,

photon - p_3^μ

e^- → p_e^μ

∴ 4vector conservation →

$$p_1^\mu + p_2^\mu = p_3^\mu + p_e^\mu$$

$$\Rightarrow \left(\frac{h}{\lambda_1}, \frac{h}{\lambda_1}, 0, 0\right) + (m_e c, 0, 0, 0)$$

$$= \left(\frac{h}{\lambda_2}, \frac{h}{\lambda_2}\cos\theta, -\frac{h}{\lambda_2}\sin\theta, 0\right) + P_e^{\mu}$$

Electron was initially at rest

$\therefore |\vec{P_e}| = 0$

$\therefore \left(\frac{E}{c}\right) = m_e c \cdot \quad (m_0 = m_e)$.

Now, $\|P_1^{\mu} + P_2^{\mu} - P_3^{\mu}\|^2 = \|P_e^{\mu}\|^2$

Isolating P_e^{μ} & taking norm both sides,

$$\Rightarrow \|P_1^{\mu}\|^2 + \|P_2^{\mu}\|^2 + \|P_3^{\mu}\|^2 + 2P_1^{\mu}\cdot P_2^{\mu} - 2P_2^{\mu}\cdot P_3^{\mu} - 2P_1^{\mu}\cdot P_3^{\mu} = \|P_e^{\mu}\|^2.$$

Note that:

$\|P_e^{\mu}\|^2 = \|P_2^{\mu}\|^2$.

and, $\|P_1^{\mu}\|^2 = \|P_3^{\mu}\|^2 = 0$

Since Photon is an E-M wave,

$\therefore$ Rest mass, $m_0 = 0$

$$\Rightarrow \cancel{P_1^{\mu}\cdot P_2^{\mu} = (P_2^{\mu} + P_1^{\mu})\cdot P_3^{\mu}}$$

$h m_e c$ ⊗

$$\Rightarrow \cancel{2}(P_1^{\mu} - P_3^{\mu})\cdot P_2^{\mu} = \cancel{2}\, P_1^{\mu}\cdot P_3^{\mu}$$

$$\left(\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\right) m_e c = \frac{h^2}{\lambda_1\lambda_2} - \frac{h^2(\cos\theta)}{\lambda_1\lambda_2}$$

$$\Rightarrow \cancel{h}\left(\frac{\lambda_2 - \lambda_1}{\cancel{\lambda_1\lambda_2}}\right) m_e c = \frac{\cancel{h^2}}{\cancel{\lambda_1\lambda_2}}(1 - \cos\theta)$$

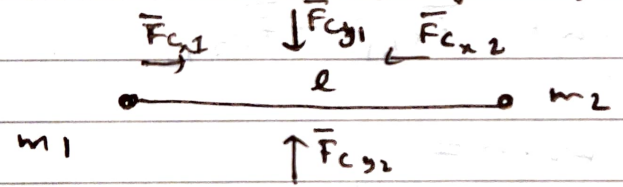
$c = 1$ $\Rightarrow$ $\boxed{\lambda_2 - \lambda_1 = \frac{h}{m_e}(1 - \cos\theta)}$ (Proved)

$\therefore$ ~~$2 \times \delta x_2 = 2 \times \delta x_1$~~ $\Rightarrow$ $\boxed{\delta x_1 = \delta x_2}$

D.

Q3) Let's say we have a rigid dumbell in 2D constraint

Forces ensure ~~rigidity~~ rigidity of dumbell



where, $\bar{F}_{c_1} = F_{c_{x1}} \hat{i} + F_{c_{y1}} (-\hat{j})$

$\bar{F}_{c_2} = F_{c_{x2}} (-\hat{i}) + F_{c_{y2}} (+\hat{j})$

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If the dumbell is static
Then,

$\Rightarrow$ $F_{Cx_1} = + F_{Cx_2}$ ie, $\bar{F}_{C_1} = -\bar{F}_{C_2}$
& $F_{Cy_1} = + F_{Cy_2}$.

Case1 Consider a translation in the plane.

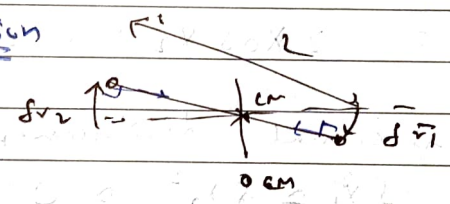


$\delta\bar{r}_1 = \delta x_1 \hat{i} + \delta y_1 \hat{j}$
$\delta\bar{r}_2 = \delta x_2 \hat{i} + \delta y_2 \hat{j}$

Now Rigidity of rod requires.
$\delta x_1 = \delta x_2$ & $\delta y_1 = \delta y_2$ $\therefore \delta\bar{r}_1 = \delta\bar{r}_2$

Then total virtual work done by constraint forces, is $\sum \bar{F}_{C_i} \cdot \delta\bar{r}_i = \bar{F}_{C_1} \cdot \delta\bar{r}_1 + \bar{F}_{C_2} \cdot \delta\bar{r}_2$
$= \bar{F}_{C_1} \cdot \delta\bar{r}_1 - \bar{F}_{C_1} \cdot \delta\bar{r}_1$
$= 0$

Case2 Rotation



Rotation about COM, (fixed)
similarly, Rigidity of rod requires
$\delta\bar{r}_1 = \delta\bar{r}_2$

Consider constraint forces along rod = $F_{C\parallel}$,
" " $\perp^r$ to rod = $F_{C\perp}$.
$\therefore \bar{F}_C = \bar{F}_{C\perp} + \bar{F}_{C\parallel}$
Note that $\bar{F}_{C_1,\perp} = -\bar{F}_{C_2,\perp}$

$\therefore$ Virtual work =
$\bar{F}_{C_1} \cdot \delta\bar{r}_1 + \bar{F}_{C_2} \cdot \delta\bar{r}_2$
$(\bar{F}_{C\parallel} \cdot \delta\bar{r}_1) + \bar{F}_{C_2,\perp} \cdot \delta\bar{r}_2 = \delta\bar{r}_2 \cdot (\bar{F}_{C_1,\perp} + \bar{F}_{C_2,\perp})$
$= 0$

($\because \delta\bar{r} \cdot \bar{F}_{C\parallel} = 0$)

Hence zero work is done in virtual displacement by constraint forces. □.