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Roll- 22MS077

PH2102, Assgn-2

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Date | 3/5/2023

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Q1) Suppose for some particle of rest mass m_0, $v = 0.5c$

(a) what % of it's mass is it's leading contribution to the KE?

(b) what is the % of the further correction if we keep the term $v^4, v^6, \ldots$

Ans

We have, $m_v = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} = m_0\left(1-(0.5)^2\right)^{-1/2} \triangleq 1.155\, m_0$.

$$m_v = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}.$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{\lfloor 2} + \frac{n(n-1)(n-2)x^3}{\lfloor 3} + \ldots$$

$$= m_0\left(1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} - \frac{5}{16}\frac{v^6}{c^6} + \ldots\right)$$

Multiplying by c^2.

$$E = m_v c^2 = m_0c^2 + \underbrace{\frac{1}{2}m_0v^2}_{\text{leading term of KE}} + \frac{3}{8}\frac{m_0v^4}{c^2} - \frac{5}{16}\frac{m_0v^6}{c^4} + \ldots$$

(a) ~~Mass contributing to~~

$$\text{Req \%} = \frac{\frac{m_0v^2}{2c^2}}{m_v}\times 100 = \left(\frac{(0.5)^2}{2}\right)\Big/1.155 \times 100$$

$$\therefore \text{Ans} = 10.82\%.$$

(b) $$\text{Req \%} = \frac{m_0\left[\frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} - \frac{5}{16}\frac{v^6}{c^6}\right]}{m_v}\times 100$$

$$= \frac{\left[(0.5)^2\times\frac{1}{2} + \frac{3}{8}(0.5)^4 - \frac{5}{16}(0.5)^6\right]}{1.155}$$

$$= 12.43\%.$$

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Q2) For a particle in motion, $E = 110$ MeV, It's rest mass is $100\ MeV/c^2$.

What is the velocity in units of c?

Ans Using relativistic Energy Formula,

$$E^2 = \frac{m_0^2 c^4}{1 - v^2/c^2}$$

$$\therefore v = c\left(1 - \frac{m_0^2 c^4}{E^2}\right)^{1/2}$$

Given $m_0 = \frac{100\ MeV}{c^2} \Rightarrow m_0 c^2 = 100 MeV$.

$$\therefore v = c\left(1 - \frac{100 \times 100}{110 \times 110}\right)^{1/2} \simeq 0.42\,c$$ Ans

Q.3) $h = 6.626 \times 10^{-34}$ Js.

$\lambda^{Na} = 5800 Å = 5800 \times 10^{-10}$ m.

Calculate the relativistic mass of the photon in free space in gm?

A- From Relativistic, Energy momentum relation

$$E^2 = |P|^2 c^2 + m_0^2 c^4.$$

For photon, rest mass $= m_0 = 0$

$$\therefore E = |P| c = h\nu$$

$$|P| = m_\gamma c \quad \therefore m_\gamma c^2 = h\nu = \frac{hc}{\lambda}$$

$\nu\lambda = c$

$\therefore$ Relativistic Mass, $= \frac{h}{c\lambda}$

$$= \frac{6.626 \times 10^{-34}}{3 \times 5800 \times 10^{-10}} \times 10^3$$

$$= 3.8 \times 10^{-33}\ g$$ Ans

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Q.4) $x' = \gamma(x - \beta ct)$
$y' = y$,
$z' = z$
$ct' = (ct - \beta x)\gamma$.

Show,
$c^2(t')^2 - (x')^2 - (y')^2 - (z')^2$
$= c^2t^2 - x^2 - y^2 - z^2$.

A ∴ $\gamma^2(ct-\beta x)^2 - \gamma^2(x-\beta ct)^2 - y^2 - z^2$

$= \gamma^2[c^2t^2 + \beta^2x^2 - 2\beta xct - x^2 - \beta^2c^2t^2 + 2\beta xct] - y^2 - z^2$

$= \gamma^2[(c^2t^2 - x^2)(1 - \beta^2)] - y^2 - z^2$

$= c^2t^2 - x^2 - y^2 - z^2$ □

Q5) Show that
$u^\mu \cdot u^\mu = 1$, where
$\left[u^\mu = \frac{1}{\sqrt{1-v^2/c^2}}, \frac{\vec{v}/c}{\sqrt{1-v^2/c^2}}\right]$

A From definition of dot product in Minkowski, 4-D space,

$u^\mu \cdot u^\mu = u^0u^0 - u^1u^1 - u^2u^2 - u^3u^3$

$= \gamma^2 - \frac{v_x^2\gamma^2}{c^2} - \frac{v_y^2\gamma^2}{c^2} - \frac{v_z^2\gamma^2}{c^2}$

$= \gamma^2\left[1 - \frac{(v_x^2 + v_y^2 + v_z^2)}{c^2}\right]$

$= \gamma^2\left[1 - \frac{v^2}{c^2}\right] = 1$ Proved

Q.6) One can similarly define 4-acceleration.

$a^\mu = \frac{du^\mu}{ds} = \frac{d^2x^\mu}{ds^2}$

Show that $u \cdot a = 0$ (in 4-D space)

$u \cdot a = u^0a^0 - u^1a^1 - u^2a^2 - u^3a^3$

Normal 3-vector velocity $\vec{v} = \sqrt{v_x^2 + v_y^2 + v_z^2}$

A Take $x^\mu = (ct, x, y, z)$.

in the frame of particle, it is at rest. So from invariance of space-time interval we have,

~~$ds^2 = c^2 dt^2 - x^2 - y^2 - z^2$~~

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 d\tau^2$$

$\therefore ds = c\, d\tau$, where τ is the proper time.

$$d\tau = dt \sqrt{1 - (dx^2 + dy^2 + dz^2)/c^2 dt^2}$$

$$= dt \sqrt{1 - \vec{v}^2/c^2} = \frac{1}{\gamma} dt.$$

$\therefore$ ~~Normal~~, 4 vector velocity,

$$u^\mu = \frac{dx^\mu}{ds} = \frac{d\tau}{ds} \frac{dt}{d\tau} \frac{dx^\mu}{dt}$$

$$= \frac{1}{c} \gamma \cdot \left(c, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$$

$$= \left(\gamma, \frac{\gamma \vec{v}}{c} \right)$$

Now,

~~Calculating~~ Calculating, $\frac{d\gamma}{dt} = \frac{d}{dt} \frac{1}{\sqrt{1 - \vec{v}^2/c^2}}$

$$= +\frac{1}{2} \frac{1}{(1 - v^2/c^2)^{3/2}} \left(\frac{+2\vec{v} \cdot}{c^2} \right) \frac{d\vec{v}}{dt}$$

$$= \frac{\gamma^3}{c^2} \vec{v} \cdot \vec{a}$$

4 vector acceleration.

$$a^\mu = \frac{du^\mu}{ds} = \frac{\gamma}{c} \cdot \frac{d}{dt} \left(\gamma, \frac{\gamma \vec{v}}{c} \right)$$

$$= \frac{\gamma}{c^2} \frac{d\gamma}{dt} (c, \vec{v}) + \frac{\gamma^2}{c^2} \frac{d}{dt} (c, \vec{v})$$

$$= \frac{\gamma^3}{c^2} \frac{\gamma}{c^2} \vec{v} \cdot \vec{a} (c, \vec{v}) + \frac{\gamma^2}{c^2} (0, \vec{a}).$$

$$= \frac{\gamma^4}{c^4} (\vec{v} \cdot \vec{a}) (c, \vec{v}) + \frac{\gamma^2}{c^2} (0, \vec{a})$$

simplifying we obtain,

$$a^\mu = \left(\frac{\gamma^4}{c^3}(\vec{v}\cdot\vec{a}),\ \frac{\gamma^4}{c^4}(\vec{v}\cdot\vec{a})\vec{v} + \frac{\gamma^2}{c^2}\vec{a} \right)$$

$$\therefore u^\mu \cdot a^\mu$$

$$= \gamma\left(\frac{\gamma^4}{c^3}\right)(\vec{v}\cdot\vec{a}) - \frac{\gamma}{c}\vec{v}\cdot\left(\frac{\gamma^4}{c^4}(\vec{v}\cdot\vec{a})\vec{v} + \frac{\gamma^2}{c^2}\vec{a}\right)$$

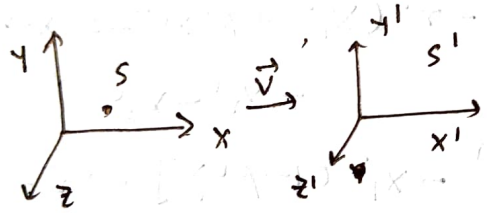
$$= \frac{\gamma^5}{c^3}(\vec{v}\cdot\vec{a}) - \frac{\gamma^5}{c^5}(\vec{v}\cdot\vec{v})(\vec{v}\cdot\vec{a}) - \frac{\gamma^3}{c^3}(\vec{v}\cdot\vec{a})$$

$$= \frac{\gamma^3}{c^3}(\vec{v}\cdot\vec{a})\left[\gamma^2 - \frac{\gamma^2 v^2}{c^2} - 1\right]$$

$$= \frac{\gamma^3(\vec{v}\cdot\vec{a})}{c^3}\left[\gamma^2\left(1 - \frac{v^2}{c^2}\right) - 1\right] = 0$$

Proved!

Q)7) Show that $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$ is Lorentz invariant.



$$\left(\frac{E}{c}\right)^2 - |P|^2 = m_0^2 c^2 > 0$$

In rest frame, S, particle is at rest.

$$E^2 = m_0^2 c^4$$

$$\Rightarrow \left(\frac{E}{c}\right)^2 = m_0^2 c^2.$$

$P_x = P_y = P_z = 0$. (say).

In frame S'

$$E' = \gamma m_0 c^2$$

$$\therefore P'_x = \gamma m_0 \bar{V}_x$$

$$P'_y = P_z = 0.$$

Am

$$= \left(\frac{E'}{c}\right)^2 - P'^2_x - P'^2_y - P'^2_z$$

$$= (\gamma m_0 c)^2 - (\gamma m_0 \bar{V})^2 \quad (-0-0)$$

$$= \gamma^2 m_0^2 c^2 - \gamma^2 m_0^2 \bar{v}^2.$$

$$= \gamma^2 m_0^2 c^2 \left(1 - \frac{\bar{V}^2}{c^2}\right)$$

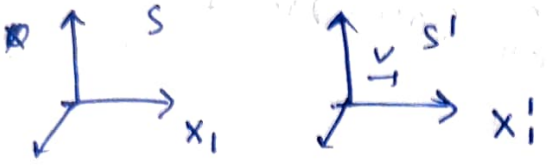
$$= m_0^2 c^2 = \left(\frac{E}{c}\right)^2 + 0 + 0 + 0$$

$$= \left(\frac{E}{c}\right)^2 - P_x^2 - P_y^2 - P_z^2$$

So, it is Lorentz Invariant.

Q. 8) show, $X^\mu X_\mu = x_0^2 - x_1^2 - x_2^2 - x_3^2$ is cont. under L-T

consider,



let, $\beta = \frac{v}{c}$.

$X^\mu = (ct, x, y, z) = (x^0, x^1, x^2, x^3)$

$x' = (x - \frac{vt}{c})\gamma$

$\therefore x^{11} = (x^1 - \beta x^0)\gamma$

$x_2' = x^2$ | $ct' = \gamma(ct - \frac{vx}{c})$

$x_3' = x^3$ | $x^{0'} = \gamma(x^0 - \beta x^1)$

$\therefore x_0'^2 + (x_1')^2 + (x^{21})^2 - (x^3)^2$

$= \gamma^2(x^0 - \beta x^1)^2 - \gamma^2(x^1 - \beta x^0)^2 - x_2^2 - x_3^2$

$= \gamma^2(x_0^2 + \beta^2 x^{12} - 2\beta x^1 x^0) - \gamma^2(x_1^2 + \beta^2 x^{02} - 2\beta x^1 x^0) - x_2^2 - x_3^2$

$= \gamma^2[x_0^2(1-\beta^2) - x_1^2(1-\beta^2)] - x_2^2 - x_3^2$

$= \gamma^2(1-\beta^2)(x_0^2 - x_1^2) - x_2^2 - x_3^2$

$= x_0^2 - x_1^2 - x_2^2 - x_3^2$ $\therefore$ Lorentz Invariant Proved. D.

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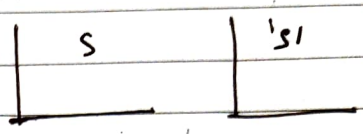
Q) consider $X^\mu = (ct, x, y, z)$

$p^\mu = (E/c, p_x, p_y, p_z)$

show that

$$X^\mu . p^\mu = ct\frac{E}{c} - xp_x - yp_y - zp_z .$$

is invariant under.



Ans Transformation Equations are

S frame	S' frame	
E, t, x, y, z; p_x, p_y, p_z.	E, t', x', y', z' p_x', p_y', p_z'	$\beta = \frac{v}{c}$

$y' = y$, $z' = z$, $p_y' = p_y$, $p_z' = p_z$.

$E' = m_0c^2\gamma$.

~~$E = \sqrt{m_0^2c^4 + (p_x^2 + p_y^2 + p_z^2)c^2}$~~

$p_x' = \gamma(p_x - \beta(E/c))$

~~$p_y' = p_y$~~

$\frac{E'}{c} = c\gamma\left(\frac{E}{c} - \beta p_x\right)$.

$x' = \gamma(x - vt)$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$.

$\therefore$ In S' frame

$$t'E' - x'p_x' - y'p_y' - z'p_z'$$

$$= \gamma\left(t - \frac{vx}{c^2}\right)c\gamma\left(\frac{E}{c} - \beta p_x\right) - \gamma^2(x - vt)\left(p_x - \frac{\beta E}{c}\right) - yp_y - zp_z .$$

$$= \gamma^2\left[\left(ct - \beta x\right)\left(\frac{E}{c} - \beta p_x\right) - (x - vt)\left(p_x - \frac{\beta E}{c}\right)\right] - yp_y - zp_z$$

$$= \gamma^2\left[Et - c\beta t p_x - \beta x\frac{E}{c} + \beta^2 x p_x - xp_x + x\frac{\beta E}{c} + p_x vt - vt\frac{\beta E}{c}\right] - yp_y - zp_z$$

$$= \gamma^2\left[Et(1 - \beta^2) - xp_x(1 - \beta^2)\right] - yp_y - zp_z$$

$= \gamma^2 (1-\beta^2)(Et - xp_x) - yp_y - zp_z$

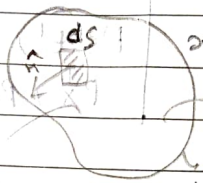
$= Et - xp_x - yp_y - zp_z$

□

10 Q) Use the continuity Eqn,

$$\frac{\partial \rho}{\partial t} + \bar{\nabla} \cdot \bar{J} = 0.$$

to show that net charge coming into a fixed volume is equal to change in charge content.



Ans ρ = density of charge,

$\bar{J}$ = current density

Consider a fixed volume, V, with boundary = ∂V.

The current flowing through a area section dS, is given by, $dI = \vec{J} \cdot \hat{n}\, dS$

∴ Net Current Flux, $I = \int_{\partial V} (\vec{J} \cdot \hat{n})\, dS$

Now this integral also represents the rate at which charge exits V ($\hat{n}$ is outside).

Also, applying Gauss's Divergence Theorem,

$$I = \int_{\partial V} (\bar{J} \cdot \hat{n})\, dS = \int_{V} (\bar{\nabla} \cdot \bar{J})\, dV$$

Now using continuity Eqn, we have,

$$I = \int_V -\frac{\partial \rho}{\partial t}\, dV = -\frac{d}{dt} \int_V \rho\, dV = -\frac{d}{dt} Q_V$$

Where Q_V = Total volume charge contained in V

So, we arrive at,

$$I + \frac{d}{dt} Q_v = 0$$

$$\Rightarrow \frac{d}{dt} Q_v + \int_{\partial V} (\bar{J} \cdot \hat{n}) \, dS = 0$$

Rate at which charge flows into a fixed volume is precisely balanced by the increase in charge content.

Q.E.D

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