PH2102 | H/W-1

2/08/2023

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(P2)

① Show equivalence of

(a) $\bar{A}\cdot\bar{B} \equiv |\bar{A}||\bar{B}|\cos\theta$ and $A_x B_x + A_y B_y + A_z B_z$.

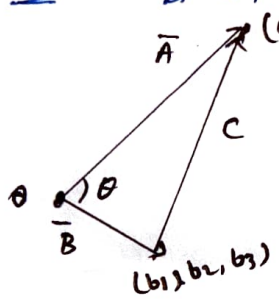
(b) $\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta\,\hat{e}$ and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$.

Ans (a) Let

$\bar{A} = \langle a_1, a_2, a_3 \rangle$

$\bar{B} = \langle b_1, b_2, b_3 \rangle$

Ip If θ is the angle between $\bar{A}$ & $\bar{B}$.



using law of cosines,

$$\cos\theta = \frac{|A|^2 + |B|^2 - |C|^2}{2|A||B|}$$

$$\Rightarrow |C|^2 = |A|^2 + |B|^2 - 2|A||B|\cos\theta. \quad \text{---} (1)$$

Now, consider,

$$|C|^2 = \bar{C}\cdot\bar{C} = (\bar{A}-\bar{B})\cdot(\bar{A}-\bar{B})$$

$$= |A|^2 + |B|^2 - 2\bar{A}\cdot\bar{B} \quad \text{---} (ii)$$

From eq (i) (ii) we compare,

$$\bar{A}\cdot\bar{B} = |A||B|\cos\theta \quad \text{---} (iii)$$

~~Proved~~

Now,

$|A| = \sqrt{a_1^2+a_2^2+a_3^2}$

$|B| = \sqrt{b_1^2+b_2^2+b_3^2}$.

$$\cos\theta = \frac{\sum a_i^2 + \sum b_i^2 - \{\sum (a_i-b_i)^2\}}{2\,(a_1^2+a_2^2+a_3^2)^{1/2}(b_1^2+b_2^2+b_3^2)^{1/2}}$$

$$\Rightarrow \cos\theta = \frac{+2(a_1b_1 + a_2b_2 + a_3b_3)}{2|A|\cdot|B|}$$

Put in eq (iii)

$$|A||B|\cos\theta = \frac{+2(\;)}{|A|\cdot|B|}|A||B| = a_1b_1 + a_2b_2 + a_3b_3$$

~~Proved~~ Shown.

② • Prove (a) $\bar{\nabla} \times (\bar{\nabla}\phi) = 0 \quad \forall\ \phi(\bar{r})$

(b) $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0 \quad \forall\ \bar{A}(\bar{r})$

From stokes' Theorem,

$$\int_S (\bar{\nabla} \times \bar{v}) \cdot d\bar{s} = \oint_\Gamma \bar{v} \cdot d\bar{r} \quad \text{over any closed loop } \Gamma.$$

subs: $\bar{v} = \bar{\nabla}\phi$.

$$\Rightarrow \int_S (\bar{\nabla} \times (\bar{\nabla}\phi) \cdot d\bar{s} = \oint (\bar{\nabla}\phi) \cdot d\bar{r} = 0.$$

for a closed loop.

since it is defined for all points on S.

$$\bar{\nabla} \times (\bar{\nabla}\phi) = 0 \quad \forall \quad \phi(\bar{r}),$$

(b) let $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.

$$(\bar{\nabla} \times \bar{A}) = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial}{\partial x}A_z - \frac{\partial}{\partial z}A_x\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial}{\partial y}A_x\right)$$

$$\therefore \bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) \equiv \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (\bar{\nabla} \times \bar{A}) = 0, \quad \because \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_z}{\partial x \partial y} + \dots$$

clearly

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(1) (b) Let us consider vectors.

$\bar{A} = \langle A_x, A_y, A_z \rangle$

$\bar{B} = \langle B_x, B_y, B_z \rangle$

and $\exists$ angle $\theta = \bar{A} \wedge \bar{B}$.

Let us consider $\hat{n}$, $\perp$ to both $\bar{A}, \bar{B}$, s.t,

~~$\hat{n} = \langle n_x \hat{i}, n_y \hat{j}, n_z \hat{k} \rangle$~~

$\hat{n} = \langle n_1, n_2, n_3 \rangle$.

Then $\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta \, \hat{n} = |A||B| \sqrt{1 - \cos^2\theta} \, \hat{n}$.

$= \sqrt{(|\bar{A}||\bar{B}|)^2 - (|\bar{A}||\bar{B}|\cos\theta)^2} \, \hat{n}$.

$= \sqrt{(|\bar{A}||\bar{B}|)^2 - (\bar{A}.\bar{B})^2} \quad \hat{n}$

$= \{ (A_x^2 B_x^2 + A_x^2 B_y^2 + A_x^2 B_z^2 + A_y^2 B_x^2 + A_y^2 B_y^2 + A_y^2 B_z^2 + A_z^2 B_x^2 + A_z^2 B_y^2 + A_z^2 B_z^2) - (A_x^2 B_x^2 + A_y^2 B_y^2 + A_z^2 B_z^2 + 2A_x B_x A_y B_y + 2A_x B_x A_z B_z + 2A_y B_y A_z B_z) \}^{1/2} \hat{n}$.

$= \{ (A_x^2 B_y^2 + A_y^2 B_x^2 - 2A_x B_x A_y B_y) + (A_x^2 B_z^2 + A_z^2 B_x^2 - 2A_x B_x A_z B_z) + (A_y^2 B_z^2 + A_z^2 B_y^2 - 2A_y B_y A_z B_z) \}^{1/2} \hat{n}$

$= \sqrt{(A_x B_y - A_y B_x)^2 + (A_x B_z - A_z B_x)^2 + (A_y B_z - A_z B_y)^2} \, \hat{n}$

$= \sqrt{\mu} \, \hat{n} \, (\text{say}) \quad \cdots (i)$

NOW,

$n_1 A_x + n_2 A_y + n_3 A_z = 0$ [As $\hat{n} \perp \bar{A}$ & $\hat{n}.\bar{A} = 0$]

& $n_1 B_x + n_2 B_y + n_3 B_z = 0$.

$\Rightarrow -n_2 \frac{A_y B_x}{A_x} - n_3 \frac{A_z B_x}{A_x} + n_2 B_y + n_3 B_z = 0$

$\rightarrow \boxed{n_1 = \frac{-n_2 A_y + A_z n_3}{A_x}}$ (ii)

$\therefore (A_x B_y - A_y B_x) n_2 = (A_z B_x - A_x B_z) n_3$

$$\Rightarrow \boxed{n_2 = n_3 \left(\frac{A_z B_x - A_x B_z}{A_x B_y - A_y B_x} \right)} \quad \text{--- (iii)}$$

Substituting (iii) in (i)

$$n_1 = n_3 \left(\frac{A_y B_z - A_z B_y}{A_x B_y - A_y B_x} \right) \quad \text{--- (iv)}$$

Now

$$\sqrt{n_1^2 + n_2^2 + n_3^2} = 1$$

$$\Rightarrow \sqrt{n_3^2 \left[\frac{(A_y B_z - A_z B_y)^2 + (A_z B_x - B_z A_x)^2 + (A_x B_y - A_y B_x)^2}{(A_x B_y - A_y B_x)^2} \right]}$$

$$\Rightarrow n_3 \frac{\sqrt{M}}{(A_x B_y - A_y B_x)} = 1$$

$$\Rightarrow n_3 = \frac{(A_x B_y - A_y B_x)}{\sqrt{M}} \quad \text{--- (v)}$$

Substituting (v) in (iii) & (iv).

$$n_2 = \frac{(A_z B_x - A_x B_z)}{\sqrt{M}} \quad \text{--- (vi)}$$

$$n_3 = \frac{(A_y B_z - A_z B_y)}{\sqrt{M}}, \quad n_1 = \frac{(A_y B_z - A_z B_y)}{\sqrt{M}} \quad \text{--- (vii)}$$

From (i)

$$\bar{A} \times \bar{B} = \sqrt{M} (n_1 \hat{i} + n_2 \hat{j} + n_3 \hat{k})$$

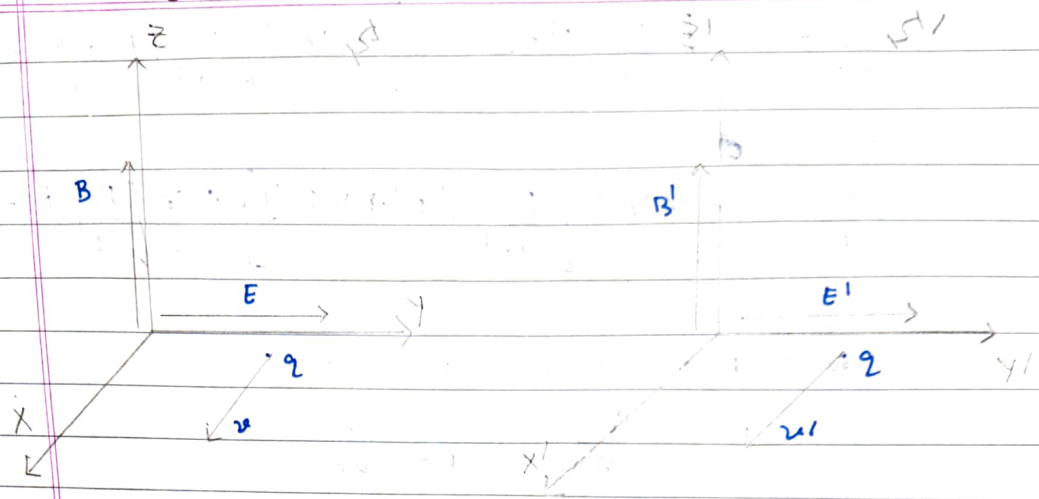
$$= (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

[subst v, vi, vii]

Thus, the two definitions of cross product are equivalent Proved

H/W-2 Lorentz Transformation



We take 2 ~~too~~ inertial frames (S, S') s.t S' moves with a velocity V wrt S along X axis.

We ~~will~~ shall show that

$$\bar{\nabla} \times \bar{B} = \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t}$$ is invariant in 2 frame under Lorentz Transformation.

We use the following coordinate transformation.

- $x' = \gamma (x - Vt)$
- $y = y'$
- $z = z'$
- $t' = \gamma \left(t - \frac{Vx}{c^2} \right)$

and,

$$E = \gamma [E' + VB']$$
$$B = \gamma \left[B' + \frac{V}{c^2} E'\right]$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

where E', B' are ~~electric~~ electric and magnetic fields experienced by q in S' frame.

In S frame, $\bar{E} = E(x,t)\,\hat{j}$

$$\bar{B} = B(x,t)\,\hat{k}.$$

$$\bar{\nabla} \times \bar{B} = -\frac{\partial B}{\partial x}\,\hat{j}.$$

So, in S eq. becomes, $\frac{\partial B}{\partial x} = \frac{-1}{c^2} \frac{\partial E}{\partial t}$.

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial B}{\partial t'} \frac{\partial t'}{\partial x}\,; \quad \frac{\partial E}{\partial t} = \frac{\partial E}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'} \cdot \frac{\partial t'}{\partial t}$$

Solving $\frac{\partial x'}{\partial x} = \gamma$, $\frac{\partial t'}{\partial x} = -\frac{\gamma V}{c^2}$, $\frac{\partial t'}{\partial t} = \gamma$, $\frac{\partial x'}{\partial t} = -V\gamma$

Substituting values,

$$\frac{\partial B}{\partial x} = \gamma\left[\frac{\partial}{\partial x'}\left[B' + \frac{v}{c^2}E'\right]\right] - \frac{\gamma v}{c^2}\left[\frac{\partial}{\partial t'}\left[B' + \frac{v}{c^2}E'\right]\right]$$

&

$$\frac{\partial E}{\partial t} = -v\gamma^2\left[\frac{\partial}{\partial x'}\left[E' + vB'\right]\right] + \gamma\left[\frac{\partial}{\partial t'}\left[E' + vB'\right]\right]$$

Putting values in eq.

$$-\frac{\partial B}{\partial x} = \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\Rightarrow -\gamma\left[\frac{\partial B'}{\partial x'} + \frac{v}{c^2}\frac{\partial E'}{\partial x'} - \frac{v}{c^2}\frac{\partial B'}{\partial t'} - \frac{v^2}{c^4}\frac{\partial E'}{\partial t'}\right]$$

$$= \frac{\gamma}{c^2}\left[-v\frac{\partial E'}{\partial x'} - v^2\frac{\partial B'}{\partial x'} + \frac{\partial E'}{\partial t'} + v\frac{\partial B'}{\partial t'}\right]$$

$$\Rightarrow -\frac{\partial B'}{\partial x'} + \frac{v^2}{c^4}\frac{\partial E'}{\partial t'} = -\frac{v^2}{c^2}\frac{\partial B'}{\partial x'} + \frac{1}{c^2}\frac{\partial E'}{\partial t'}$$

$$\Rightarrow -\frac{\partial B'}{\partial x'}\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{c^2}\left(1 - \frac{v^2}{c^2}\right)\frac{\partial E'}{\partial t'}$$

$$\Rightarrow \boxed{\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'}}$$

Thus, shown. Equation is invariant □.

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Q1) Given the eqn,

$a_{11}^2 (x - vt)^2 - c^2 (a_{41} x + a_{44} t)^2 = 0$

& $x^2 - c^2 t^2 = 0$

Deduce the value of a_{11}, a_{41}, a_{44}

Ans Coeff of t^2, x^2, xt are →

- $a_{11}^2 v^2 - c^2 a_{44}^2 = -c^2$ — (i)
- $a_{11}^2 - c^2 a_{41}^2 = 1$ — (ii)
- $-a_{11}^2 v - c^2 a_{41} a_{44} = 0$ — (iii)

From eq (i), (ii), +!

$a_{44}^2 = \frac{c^2 (a_{11}^2 \frac{v^2}{c^2} + 1)}{c^2}$

take $\beta = \frac{v}{c}$

$= a_{11}^2 \beta^2 + 1$

$\therefore a_{41}^2 = \frac{(a_{11}^2 - 1)}{c^2}$

Sq. eq (iii) both sides & sub. val.

$a_{11}^4 v^2 = c^2 \left(\frac{a_{11}^2 - 1}{c^2}\right)\left(a_{11}^2 \beta^2 + 1\right)$

$a_{11}^4 \beta^2 = a_{11}^4 \beta^2 + a_{11}^2 - a_{11}^2 \beta^2 - 1$

$\therefore a_{11} = \frac{\pm 1}{\sqrt{1-\beta^2}}$ --- (iv)

$\therefore a_{41}^2 = \frac{1/1-\beta^2 - 1}{c^2}$

$= \frac{v^2/c^4}{1 - \frac{v^2}{c^2}}$

$a_{41} = \pm \frac{v/c^2}{\sqrt{1 - v^2/c^2}}$ --- (v)

we find, $a_{44} = \pm \frac{1}{\sqrt{1-\beta^2}}$

(vi)

checking consistency of our solution.

using eq (ii)

$a_{11}^2 v = -c^2 a_{41} a_{44} > 0$

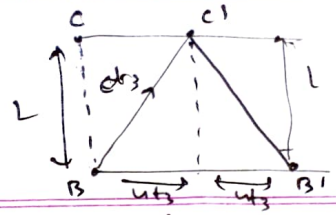
$\Rightarrow c^2 a_{41} a_{44} < 0$

$\Rightarrow$ They must be of opp. signs.

If we Take, $a_{11} = \frac{+1}{\sqrt{1-\beta^2}} = a_{44}$

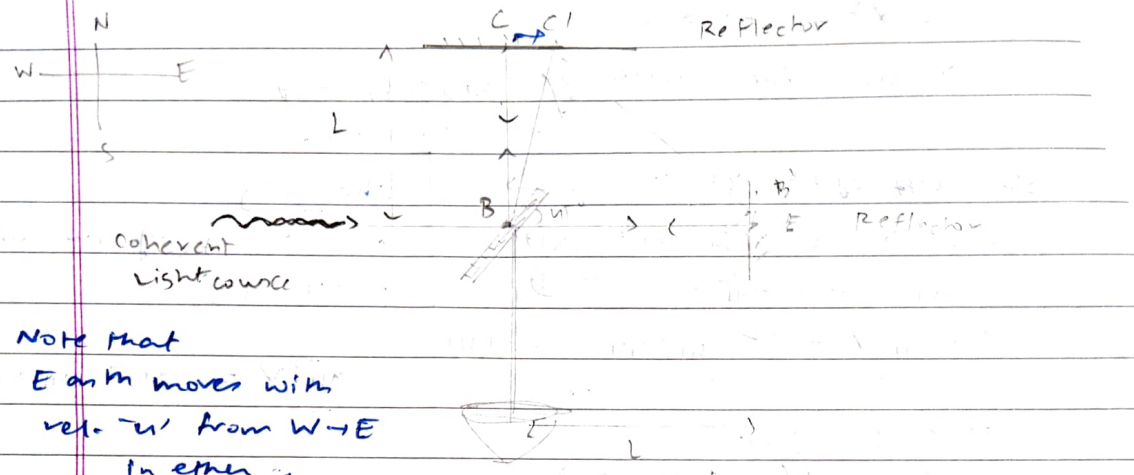
Then $a_{41} = \frac{-v/c^2}{\sqrt{1 - v^2/c^2}}$.

when, $\beta = \frac{v}{c}$.



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Q) Prove that Lorentz contraction, explains Michelson-Morley experiment.



Note that
Earth moves with
vel. u from W→E
In ether :

Ether - Frame - S
Earth - Frame - S'

Time taken by light to travel B E B = $t_1 + t_2$

$ct_1 = L' + t_1 u$

$ct_2 = L' - t_2 u$.

$\therefore t_1 = \frac{L}{c-u}$, $t_2 = \frac{L}{c+u}$.

$\therefore t_1 + t_2 = \frac{2L'/c}{1-\frac{u^2}{c^2}}$.

$L' \to S'$
$L \to S$.

Now, due to L.C.

$L' = L\sqrt{1-\frac{u^2}{c^2}}$.

$\therefore t_1 + t_2 = \frac{2L/c}{\sqrt{1-\frac{u^2}{c^2}}}$ ... (1)

Time taken to travel, B C1 B1 $= 2t_3$

$(ct_3)^2 = L^2 + u^2 t_3^2$.

$t_3^2 (c^2 - u^2) = L^2$.

$t_3 = \frac{L}{\sqrt{c^2-u^2}}$.

$\therefore 2t_3 = \frac{2L/c}{\sqrt{1-u^2/c^2}}$

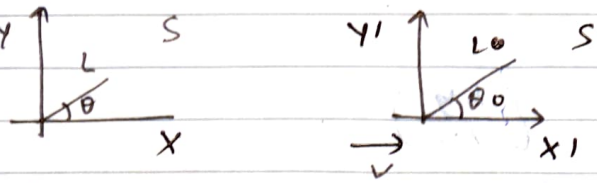
Thus we find $t_1 + t_2 = 2t_3$
despite motion of Earth.

Hence, we observe No. Fringe shift.

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Q)



S' is moving with vel. V, along +X axis wrt S.

(a) Find length of rod wrt S & frame.

(b) Find the Angle θ in S?

A (a) $L'_y = L_0 \sin\theta_0$

$L'_x = L_0 \cos\theta_0$.

L'_x will only be Lorentz contracted.

Observer in S, o' will measure both ends of the rod simultaneously.

$\therefore L_y = L'_y$ Transformation from frame S' to S.

$x' = (x - vt)\gamma$

$\frac{x_1'}{\gamma} = x_1 - vt_1$

& $\frac{x_2'}{\gamma} = x_2 - vt$.

$\therefore \frac{\Delta x'}{\gamma} = \Delta x$

$\therefore L_x = \frac{L'_x}{\gamma} = L_0 \cos\theta_0 \sqrt{1 - \frac{v^2}{c^2}}$.

$\therefore$ Length of rod in S frame is.

$$l = \sqrt{L_x^2 + L_y^2} = L_0 \sqrt{\sin^2\theta_0 + \cos^2\theta_0 \left(1 - \frac{v^2}{c^2}\right)}$$

$$= L_0 \left(1 - \cos^2\theta \frac{v^2}{c^2}\right)^{1/2}$$ Ans

(b)

New angle,

$$\theta = \tan^{-1}\left(\frac{L_y}{L_x}\right)$$

$$= \tan^{-1}\left(\frac{L_0 \sin\theta_0}{L_0 \cos\theta_0 \sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\theta = \tan^{-1}\left(\frac{\tan\theta_0}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$ Ans

(Q) A light beam passes through a horizontal column of flowing water (vel = v)



n = Refractive Index of medium.

(i) What is the speed of light from the lab frame wrt observer? = u

(ii) Obtain a u in the first order approximation, (~v/c)

(i) using the velocity addition,

$$u = \frac{u' + V}{1 + \frac{Vu'}{c^2}}$$

Now V = velocity of water.

$u' = \frac{c}{n}$ (∵ Refraction)

∴ speed of light, as observed from lab frame.

$$u = \frac{\frac{c}{n} + V}{1 + \frac{V}{c^2}\cdot\frac{c}{n}} = \frac{\frac{c}{n} + V}{1 + \frac{V}{cn}}$$ Ans

(ii) For 1st Order approximation.

$$u = \left(\frac{c}{n} + v\right)\left(1 + \frac{v}{cn}\right)^{-1}$$

or

$$(1+x)^{-1} = 1 - x + x^2 - x^3 \cdots$$

$$\therefore u = \left(\frac{c}{n} + v\right)\left(1 - \frac{v}{cn} + \frac{v^2}{c^2n^2} \cdots\right)$$

Neglecting higher order term.

$$u = \frac{c}{n} + v - \frac{v}{cn}\left(\frac{c}{n} + v\right)$$

$$= \frac{c}{n} + v - \frac{v}{n^2} + \frac{v^2}{cn}$$

$$\therefore u = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)$$